TOPIC:"INDEFINITE INTEGRATION"

1. If $x = \phi(t)$ is a differentiable function of "t" then prove that $\int f(x)\,dx = \int f[\phi(t)]\,\phi'(t)\,dt$.
2. Prove the following (a) $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C, IFn \neq -1$

   (b) $\int \frac{f'(x)}{f(x)}\,dx = \log |f(x)| + c$

   (c) $\int \frac{f'(x)}{\sqrt[n]{f(x)}}\,dx = \frac{[f(x)]^{1-\frac{1}{n}}}{1-\frac{1}{n}} + c, n \neq 1$
3. Prove that :

   $\int e^x [f(x) + f'(x)]dx = e^x . f(x) + c$ and hence find $\int e^x (\cot x - \operatorname{cosec}^2 x)dx$
4. If u and v are differentiable function of x, then prove that :

   $\int u\,v\,dx = u\int v\,dx - \int \left[\frac{du}{dx}.\int v\,dx\right]dx$
5. Prove that $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x + \sqrt{x^2+a^2}\right) + c$
6. Prove that $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log |x + \sqrt{x^2-a^2}| + c$
7. Prove that $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
8. Evaluate: $\int \frac{dx}{\sin(x-a)\sin(x-b)}$
9. Evaluate: $\int \sin^3 x \cos x\,dx$
10. Evaluate: $\int \log x\,dx$
11. Evaluate : $\int \frac{dx}{x+\sqrt{x}}$
12. Evaluate : $\int \frac{dx}{x+\sin x}$
13. Evaluate : $\int \sin(\log x)dx$
14. Evaluate : $\int \frac{x\,dx}{(x-1)(x^2+1)}$

15. Evaluate: $\int \frac{\log x}{(1+\log x)^2}\, dx$

16. Evaluate: $\int \frac{1}{x(x-1)}\, dx$

17. $\int \sec^n x \tan x \, dx$

18. Evaluate: $\int \frac{dx}{\sin^2 x + 5\sin x \cos x + 2}$

19. Evaluate: $\int \frac{\log x}{(1+\log x)^2} dx$

20. Integrate $\sec^3 x$ w .r .t x

21. Evaluate : $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x} dx$

22. Evaluate : $\int \frac{dx}{\cos x(2+\sin x)}$

23. Evaluate: $\int e^x \frac{\log(\sin e^x)}{\tan e^x} dx$

24. Evaluate: $\int \frac{2x}{(x+5)^2} dx$

25. Evaluate: $\int \cos\sqrt{x}\, dx$

26.Evaluate : $\int \frac{dx}{\sqrt{x^2+6x+5}}$

27.Evaluate : $\int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}}.dx$

28. Evaluate: $\int x \log x \, dx$

29. Evaluate: $\int \frac{1}{5+4\cos x} dx$

30. Evaluate: $\int \frac{1}{a^2 e^x + b^2 e^{-x}} dx$

31. Evaluate: $\int \frac{\sin(x-a)}{\sin x} dx$

32. Evaluate: $\int \frac{x\,dx}{(x-1)(x^2+1)}$

33. Evaluate : $\int \frac{\sin(x-a)}{\sin(x+a)}dx$

34. Evaluate : $\int \frac{x\,dx}{(x+1)^2(x+2)}$

35. Evaluate: $\int \frac{x^2}{(x^2+2)(2x^2+1)}\,dx$

36. Evaluate : $\int \frac{dx}{\sqrt{x}+x}$

37. Evaluate : $\int x\sec^2 x\,dx$

38. Evaluate : $\int \frac{1}{3+5\cos x}\,dx$

39. Evaluate : $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+a)}}$

40. Evaluate : $\int \frac{e^x}{\sqrt{2e^{2x}+7e^x-5}}dx$

41. Evaluate : $\int \frac{x^2+37}{(x^2-7)\,(x^2+4)}dx$

42. Evaluate : $\int \frac{5x+2}{x^2-3x+2}dx$

43. Evaluate : $\int \frac{dx}{\sin^2 x.\cos^2 x}$

44. Evaluate : $\int \operatorname{cosec}^3 x\,dx$

45. Evaluate : $\int \frac{dx}{3\sin x+4\cos x+5}$

46. Evaluate : $\int \frac{dx}{2x+3x\log x}$

47. Evaluate : $\int \frac{\sec^2(\log x)}{x} dx$

48. Evaluate : $\int \frac{dx}{1+\cos 2x}$

49. Evaluate : $\int \frac{\sin x}{\cos(x-a)} dx$

50. Evaluate : $\int \log x \, dx$

51.Evaluate : $\int \frac{dx}{(x-1)^2(x+1)}$

52. $\int \frac{1}{x}.\log x \, dx = ...$

(a) log (log x) +c (b) $\frac{1}{2}(\log x)^2 + c$ (c) $2\log x + c$ (d) $\log x + c$

53. $\int \frac{1}{1+\cos x} dx = ...$

(a) $\tan\left(\frac{x}{2}\right) + c$ (b) $2\tan\left(\frac{x}{2}\right) + c$ c) $-\cot\left(\frac{x}{2}\right) + c$ (d) $-2\cot\left(\frac{x}{2}\right) + c$
