TOPIC: "DIFFERENTIATION"

1. If $y = f(x)$ and $x = g(y)$ where 'g' is the inverse function of f and if $\dfrac{dy}{dx}$ and $\dfrac{dy}{dx}$ both exits and $\dfrac{dy}{dx} \neq 0$,then prove that $\dfrac{dy}{dx} = \dfrac{1}{\left(\dfrac{dx}{dy}\right)}$.

2. Prove that if a function $f(x)$ is differentiable at $x = a$ then it is continuous at $x = a$.

3. If $y = f(x)$ is a differentiable function of 'u' and $u = g(x)$ is a differentiable Function of 'x', then prove that $y = f[g(x)]$ is differentiable function of 'x' and $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$.

4. If $x = f(t), y = g(t)$ are differentiable function of parameter $'t'$ then prove that y is a differentiable function of 'x' and $\dfrac{dy}{dx} = \dfrac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)}, \dfrac{dx}{dt} \neq 0$, Hence find $\dfrac{dy}{dx}$ if $x = a\cos t, y = a\sin t$.

5. If y is a differentiable function of u, and u is a differentiable function of x ,then Show that,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} .$$

6. Prove that every differentiable function is continuous.

7. Show that the function $f(x) = |x|$ is not differentiable at the point $x = 0$.

8. If $y = e^{ax}$, show that $x\dfrac{dy}{dx} = y \log y.$

9. If $y = \left(\tan^{-1} x^2\right)$, show that $\left(1 + x^2\right)^2 \dfrac{d^2 y}{dx^2} + 2x\left(1 + x^2\right)\dfrac{dy}{dx} - 2 = 0$.

10. If $\log_{10}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = 2$, then show that $\dfrac{dy}{dx} = \dfrac{-99x^2}{101y^2}$.

11. If $x^p y^q = (x + y)^{p+q}$ then prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$.

12. If $\sec\left(\dfrac{x + y}{x - y}\right) = a^2$, then $\dfrac{d^2 y}{dx^2} = \ldots$

      (a) y (b) x

      (c) $\dfrac{y}{x}$ (d) 0

13. If $y = \sin^{-1}(3x) + \sec^{-1}\left(\dfrac{1}{3x}\right)$, find $\dfrac{dy}{dx}$.

14. If $x^y = e^{x-y}$, then $\dfrac{dy}{dx} = \ldots$

      (a) $\dfrac{1 + x}{1 + \log x}$ (b) $\dfrac{\log x}{(1 + \log x)^2}$

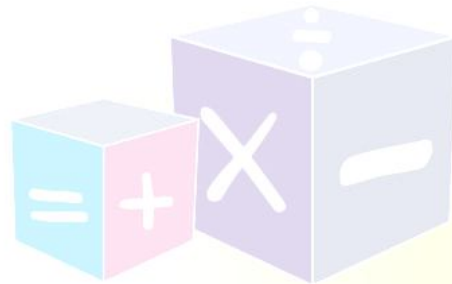   (c) $\dfrac{1 - \log x}{1 + \log x}$ (d) $\dfrac{1 - x}{1 + \log x}$.

15. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$ show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1 - y^2}{1 - x^2}}$.

16. If $y = 1 - \cos\theta, x = 1 - \sin\theta$, then $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{4}$ is ....

      (a) -1   (b) 1

      (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{2}}$

17. If $y = \tan^{-1}\left(\dfrac{5x + 1}{3 - x - 6x^2}\right)$, show that $\dfrac{dy}{dx} = \dfrac{1}{1 + (3x + 2)^2} + \dfrac{2}{1 + (2x - 1)^2}$.

18. If 2y = $\sqrt{x+1} + \sqrt{x-1}$, show that $4(x^2-1)\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} - y = 0$.

19. Find the derivative of $x\sin x$ with reference to x by first principle .

20. If $y = \sqrt{x}$; find $\dfrac{dy}{dx}$ using first principles.

21. Find derivative of $\dfrac{1}{x^2+4}$ by using first principle .

22. If $x = at^2, y = 2at$, then find $\dfrac{dy}{dx}$.

23. If $x = a\left(t - \dfrac{1}{t}\right), y = a\left(t + \dfrac{1}{t}\right)$ then show that $\dfrac{dy}{dx} = \dfrac{x}{y}$.

24. If $ax^2 + 2hxy + by^2 = 0$, show that $\dfrac{d^2y}{dx^2} = 0.$

25. Differentiate x^5 with respect to 5^x .

26. Find $\dfrac{dy}{dx}$, if $y = \log_2 x + \log_x x$.

27. If $y = \left(\sin^{-1} x\right)^2$, show that, $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = 2$.
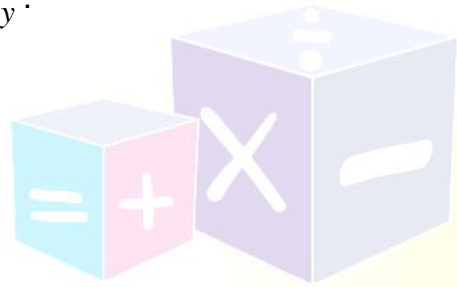
28. Find $\dfrac{dy}{dx}$, if $y = (\tan x)^x + (4)^{\sin x}$.

29. Differentiate $\left(x^x + a^a\right)$ with respect to x.

30. Find $\dfrac{dy}{dx}$,if $y = \tan(xe^x)$.

31. If $(x^2 + y)^{17} = x^8 y^{13}$,Prove that $\dfrac{dy}{dx} = \dfrac{2y}{x}$.

32. If $y = \cot^{-1}\left(\dfrac{1+15x^2}{2x}\right)$,then find $\dfrac{dy}{dx}$.

33. If $\sin y = x \sin(a + y)$, then show that $\dfrac{dy}{dx} = \dfrac{\sin^2(a + y)}{\sin a}$.

34. Differentiate 7^x with respect to $\log_7 x$.

35. If $y = \tan(4\sqrt{x} + 5)$, find $\dfrac{dy}{dx}$.

36. Find $\dfrac{dy}{dx}$ if $\sin^{-1}\left(\dfrac{2x}{1 + x^2}\right)$.

37. If $y = \dfrac{(\tan x)^x}{1 + x^2}$,find $\dfrac{dy}{dx}$.

38. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$, show that, $\dfrac{dy}{dx} = \dfrac{\cos x}{2y - 1}$.

39. If $x^y = 3^{x-y}$ show that, $\dfrac{dy}{dx} = \dfrac{x \log 3 - y}{x \log 3x}$.

40. Differentiate $\operatorname{cosec}^{-1}\left(\dfrac{1}{\sqrt{1 - x^2}}\right)$ with respect to x.

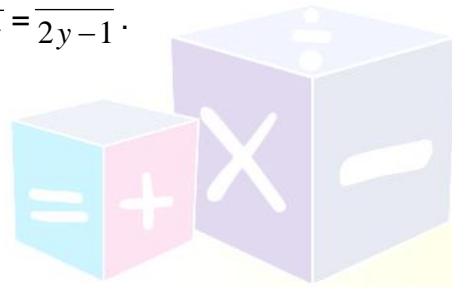41. If $y = \sin x^0 + \cos x^0$,find $\dfrac{dy}{dx}$.

42. Differentiate $\log\left[2^x\left(\dfrac{x - 1}{x + 1}\right)^{\frac{5}{2}}\right]$ with respect to x.

43. If $\sin y = \log(x + y)$, then show that $\dfrac{dy}{dx} = \dfrac{1}{(x + y \cos y - 1)}$.

44. If $y = \sin(x + y)$, find $\dfrac{dy}{dx}$.

45. If $y = \cos^{-1}(4x^3 - 3x)$,find $\dfrac{dy}{dx}$.

46. If $y = \log\left(\dfrac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2} - x}\right)$; find $\dfrac{dy}{dx}$.

47. If $y = x.e^{xy}$ then show that, $\dfrac{dy}{dx} = \dfrac{y(1+xy)}{x(1-xy)}$.

48. Differentiate $\sin^{-1}\left(\dfrac{2x}{1+x^3}\right)$ w .r .t. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$.

49. If $y = e^{m\tan^{-1}x}$ then show that, $(1+x^2)\dfrac{d^2y}{dx^2} + (2x-m)\dfrac{dy}{dx} = 0$.

50. Find $\dfrac{dy}{dx}$ if $y = \log(e^x \sin^5 x)$.

51. If $y = e^{mx} + e^{-mx}$, prove that $\dfrac{d^2y}{dx^2} = m^2 y$

52. If $x = \cos(xy)$, find $\dfrac{dy}{dx}$.

53. Differentiate, $\sin^{-1}\left(\dfrac{5\sin x + 4\cos x}{\sqrt{41}}\right)$ w. r . t. x .

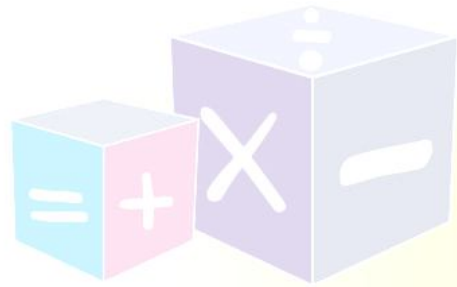54. Find $\dfrac{dy}{dx}$, if $x^y = 2^{x-y}$.

55. Find $\dfrac{dy}{dx}$; if $y = \tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$.

56. If $x^y = e^{x-y}$ show that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.

57. If $y = \sec^{-1}\sqrt{x}$, then show that $(2x\sqrt{x-1})\dfrac{dy}{dx} = 1$.

58. Differentiate with respect to x, $\sin^{-1}\left(\dfrac{a\cos x + b\sin x}{\sqrt{a^2 + b^3}}\right)$.