MATHEMATICS PAPER-9

General Instruction:

1. **All questions are compulsory.**
2. **This question paper contains 29 questions.**
3. **Question 1-4 in Section A are very short answer type questions carrying 1 mar each.**
4. **Questions 5-12 in Section B are short answer type questions carrying 2 marks each.**
5. **Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.**
6. **Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.**
7. **There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.**

Time- 3 Hours **[Max. Marks-100]**

SECTION -A

1. Find the area of the parallelogram determined by the vectors $\hat{i}+2\hat{j}+3\hat{k}\ and\ 3\hat{i}-2\hat{j}+\hat{k}$.

2. If $y=\tan^{-1}\left(\frac{a+x}{1-ax}\right), find\ \frac{dy}{dx}$.

3. If $A=\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$, show that A^{-1} does not exist.

4. If $\int_0^1 \left(3x^2+2x+K\right)dx=0$, find K.

SECTION - B

5. Evaluate $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x dx$.

6. Show that the function $f(x)=\begin{cases} 1+x, & if\ x\le 2 \\ 5-x, & if\ x>2 \end{cases}$, is not differentiable at x=2.

7. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

8. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}\ and\ \vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}, where\ \vec{a}\ne\vec{d}\ and\ \vec{b}\ne\vec{c}$.

9. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. Find the probability that the problem will be solved.

10. Examine the continuity of $f(x)=\begin{cases}\dfrac{\log x-\log 2}{x-2}, & x>2\\ \dfrac{1}{2}, & x=2 at\ x=2\\ 2\left(\dfrac{x-2}{x^2-4}\right), & x<2\end{cases}$

11. Evaluate the determinant $\Delta=\begin{vmatrix}\log_3 512 & \log_4 3\\ \log_3 8 & \log_4 9\end{vmatrix}$.

12. Differentiate $\tan^{-1}\left(\dfrac{1+2x}{1-2x}\right)$ with respect to $\sqrt{1+4x^2}$.

SECTION - C

13. Form the differential equation of the family of hyperbolas having foci on Y-axis centre at origin.

14. Evaluate $\int\dfrac{dx}{\sin(x-a)\cos(x-b)}$.

OR

Evaluate $\int\dfrac{xe^{2x}}{(1+2x)^2}dx$.

15. Two bikers are running at the speed more than speed allowed on the road along lines $\vec{r}=\left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}+\hat{k}\right) and\ \vec{r}=\left(-\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(7\hat{i}-6\hat{j}+\hat{k}\right)$.Using shortest distance, check whether they meet to an accident or not.

16. Let X denotes the number of hours, you study during a randomly selected school days. The probability that X can take the values x has the following form, where k is any unknown constant.

$$P(x)=\begin{cases}0.1, & if\ x=0\\ kx, & if\ x=1 or 2\\ k(5-x), & if\ x=3 or\ 4\\ 0, & otherwise\end{cases}$$

(i) Find the value of k

(ii) What is the probability that you study (a) atleast 2 h? (b) exactly 2 h?

17. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution and mean of numbers of tails.

18. If y(x) is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x \; and \; y(0)=1,$ find the value of $y\left(\frac{\pi}{2}\right)$.

19. If $a_1, a_2, a_3, ...a_r$ are in G.P. Prove that the determinant $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$ is independent of r.

OR

Evaluate $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$.

20. If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\frac{1}{\sqrt{3}},$ then find the value of x.

21. Evaluate $\int_0^{2\pi} \frac{x\sin^{2n}}{\sin^{2n} x + \cos^{2n} x} dx$

OR

For $x > 0$, let $f(x) = \int_1^x \frac{\log_e t}{1+t} dt.$ Find the function $f(x) + f\left(\frac{1}{x}\right)$ and show that $f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$.

22. If $\vec{a}, \vec{b} \; and \; \vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}\left[\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}\right]$ gives, deduce the condition that the three points $\vec{a}, \vec{b} \; and \; \vec{c}$ are collinear. Also, find the unit vector normal to the plane of the triangle.

23. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

SECTION - D

24. If R_1 and R_2 be two equivalence relations on a set A, Prove that $R_1 \cap R_2$ is also an equivalence relation on A.

OR

Let X be a non-empty set and P(X) be its power set. Let '*' be an operation defined on elements of P(X) by $A * B = A \cap B, \forall A, B \in P(X)$. Then

(i) Prove that '*' is a binary operation in p(X).
(ii) Is * commulative?
(iii) Is * associative?

(iv) Find the identity element in P(X) w.r.t. ‘*’.

(v) Find all the invertiable elements of P(X).

(vi) If O is another binary operation defined on P(X) as $AoB = A \cup B,$ then verify that O distributes itself over ‘*’.

25. Find the intervals in which the function given by $f\left(x\right)=\dfrac{4\sin x-2x-x\cos x}{2+\cos x}, 0\le x\le 2\pi$ is

(i) Strictly increasing and

(ii) strictly decreasing

26. A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 calories.

Two foods A and B are available at cost of `4 and `3 per unit, respectively 1 unit of food A contains 200 units of vitamins, 1 unit off minerals and 40 calories. Food B contains 100 units of vitamins, 2 units of minerals and 40 calories.

Find what combination of food should be used to have the least cost. Why a proper diet is required for us?

27. Find $A^{-1}, if\ A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ and show that $A^{-1}=\dfrac{A^2-3I}{2}$.

OR

If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}, find\ A^{-1}$ and hence solve the system of linear equation $x+2y+z=4, -x+y+z=0, x-3y+z=2.$

28. Find the distance of the point $\left(-2,3,-4\right)$ from the line $\dfrac{x+2}{3}=\dfrac{2y+3}{4}=\dfrac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$

OR

Find the coordinates of foot of perpendicular drawn from the point (0, 2, 3) on the line $\dfrac{x+3}{5}=\dfrac{y-1}{2}=\dfrac{z+4}{3}$. Also, find the length of perpendicular.

29. Using integration, find the area off the region between the circles $x^2+y^2=4\, and\left(x-2\right)^2+y^2=4$.