MATHEMATICS **PAPER-8**

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. If $y=\sin^{-1}(\sin x), x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$, then evaluate $\frac{dy}{dx}$.

2. If area of a triangle with vertices $(k,0),(1,1)\, and\, (0,3)$ is 5sq units, find the value(s) of k.

3. Find all the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i}+2\hat{j}+\hat{k}\, and -\hat{i}+3\hat{j}+4\hat{k}$.

4. Evaluate $\int_{-\pi/6}^{\pi/6} x^3\cos^2 x dx.$

SECTION - B

5. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, prove that $\vec{a},\vec{b}\, and\, \vec{c}$ are coplanar.

6. Verify Lagrange's mean value theorem for $f(x)=\log x\, in\, [1,2]$.

7. The total revenue (in `) received from the sale of x units of a product is given by $R(x)=13x^2+26x+15$. Find the marginal revenue when x=7.

8. If the function $f(x)=\frac{1}{x+2}$, find the points of discontinuity of the composite function $y=f(f(x))$.

9. Using elementary transformations, find the inverse of the matrix $\begin{bmatrix}3 & 10\\ 2 & 7\end{bmatrix}$, if it exists.

10. If $x\sqrt{1+y}+y\sqrt{1+x}=0\, and\, x\neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$.

11. Find the probability of drawing a diamond card in each of the two consecutive draws from a well-shuffled pack of cards, if the card drawn is not replaced after the first draw.

12. Evaluate $\int_{\pi/4}^{\pi/2} \sqrt{1-\sin 2x}\,dx.$

SECTION - C

13. Evaluate $\int \frac{\sin x}{\sqrt{1+\sin x}}\,dx.$

14. If $x, y, z \in [-1,1]$, such that $\sin^{-1} x+\sin^{-1} y+\sin^{-1} z=\frac{-3\pi}{2}$, find the value of $x^2+y^2+z^2$.

OR

Prove that $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.

15. Using properties of determinants, prove that following $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}=\left(1+a^2+b^2\right)^3$.

OR

If x, y, z are in G.P. using properties of determinants, show that $\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix}=0$,

where $x \neq y \neq z$ and p is any real number.

16. Evaluate $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx.$

OR

Evaluate $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx.$

17. From the differential equation of the family of circles touching the X-axis at origin.

18. Solve $\left(x^3-3xy^2\right)dx=\left(y^3-3x^2y\right)dy$.

19. By computing shortest distance, determine whether the following pair of lines intersect or not. $\vec{r}=\left(4\hat{i}+5\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and $\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(2\hat{i}+4\hat{j}-5\hat{k}\right)$

20. Profit function of a company is given as $P(x)=\frac{24x}{5}-\frac{x^2}{100}-500$, where x is the number of units produced. What is the maximum profit of the company? Company feels its social responsibility and decided to contribute 10% of his profit for the orphanage. What is the amount contributed by the company for the charity? Justify that every company should do it.

21. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of number of ace cards.

22. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be non-zero, non-coplanar vectors. Prove that a $\vec{a} - 2\vec{b} + 3\vec{c}, -2\vec{a} + 3\vec{b} - 4\vec{c}$ *and* $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar vectors.

23. In a hockey match, both teams A and B scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternatively and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respectively probabilities of winning the match.

SECTION - D

24. Let * be the binary operation on N given by a*b =LCM of a and b. Find

(i) 5 * 7, 20 * 16

(ii) Is * commutative?

(iii) Is * associative?

(iv) Find the identity of * in N.

(v) Which elements of N are invertible for the operation *?

OR

Let N denote the set of all natural numbers and R be the relation on N×N defined by d(a, b) R(c, d), if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

25. Find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$, by using integration.

26. Find the image of line $\frac{x-1}{3} = \frac{y-1}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$.

OR

Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

27. David wants to invest atmost `1200 in bonds a and B. According to the rule, he has to invest atleast `2000 in bond A and atleast `4000 in bond B. If the rates of interest on bonds A and B respectively are 8% and 10% per annum. Formulate the problem as linear programming problem and solve it graphically for maximum interest. Also, determine the maximum interest received in a year. Why investment is important for further life?

28. Find the equations of tangent and normal to the curve $y = \frac{(x-7)}{(x-2)(x-3)}$ at the point, where it cut the X-axis.

OR

Show that the equation of normal at any point on the curve $x = 3$

$\cos\theta-\cos^2\theta, y=3\sin\theta-\sin^3\theta$ is $4\left(y\cos^3\theta-x\sin^3\theta\right)=3\sin 4\theta$.

29. If $A=\begin{bmatrix} 0 & \tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ *and I* is the identity matrix of order 2, show that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.