

General Instruction:

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Question 1-4 in Section A are very short answer type questions carrying 1 mar each.
4. Questions 5-12 in Section B are short answer type questions carrying 2 marks each.
5. Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.
6. Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.
7. There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

Time- 3 Hours

[Max. Marks-100]

SECTION - A

1. Evaluate $\int_{-1}^1 x|x| dx$.
2. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, find $[A+2B]'$.
3. Differentiate $\tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ with respect to $\sqrt{1+4x^2}$.
4. Write the Cartesian equation of the line $\vec{r} = (2\hat{i} + \hat{j}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$.

SECTION - B

5. If A and B are square matrices such that $B = -A^{-1}BA$, then find the value of $(A+B)^2$.
6. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+3}{-2}$ are perpendicular.
7. If $y = \sin^{-1}(\sqrt{x}\sqrt{1-x^2} - x\sqrt{1-x})$, then find $\frac{dy}{dx}$.
8. Find a point on the parabola $y = (x-3)^2$ where the tangent is parallel to the chord joining (3,0) and (4,1).
9. The objective of A diet problem is to ascertain the quantities of certain foods that should be eaten to meet certain nutritional requirement at minimum cost. The consideration is limited to milk, beef and eggs, and to vitamins A, B, C. The number of milligrams of each of these vitamins contained within A unit of each food is given below

Vitamin	Litre of	Kg of beef	Dozen of eggs	Minimum daily
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	milk			requirements
A	1	1	10	1 mg
B	100	10	10	50 mg
C	10	100	10	10mg
Cost	₹1.00	₹1.10	₹0.50	

What is the linear programming formulation for this problem?

- Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x coordinate is 3.
- Find the differential equation of the family of all straight lines passing through the origin.
- Three cards are drawn successively without replacement from a pack of 52 well-shuffled cards. What is the probability that first two cards are king and the third card drawn is an ace?

SECTION - C

- Show that $f(x) = |x-2| + |x-3|$ is not differentiable at $x = 2$.
 - Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.
 - A trust fund has ₹30000 is to be invested in two different types of bonds. The first bond pays 9% interest per annum which will be given to orphanage and second bond pays 11% interest per annum which will be given to an NGO cancer aid society. Using matrix multiplication, determine how to divide ₹30000 among two types of bonds, if the trust fund obtains an annual total interest of ₹3060? What are the values reflected in the question?
 - Prove that $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \left(\frac{\pi}{4} + \frac{x}{2}\right), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
 - Find the differential equation of the family of circles in the first quadrant which touch the coordinate axes.
 - Evaluate $\int_0^{\pi/2} \frac{\sin^{4/5} x}{\cos^{4/5} x + \sin^{4/5} x} dx$.
- OR
- Evaluate integrals as a limit of sum $\int_2^4 2^x dx$.
- Evaluate $\int (x+1)\sqrt{1-x-x^2} dx$.
 - The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$, respectively. Assuming that the events, 'A.
 - A fruit grower can use two types of fertilizers in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are

given in the table. Tests indicate that the garden needs atleast 240 kg of phosphoric acid, atleast 270 kg of potash and almost 310 kg of chlorine. If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

22. Find the equation of the plane passing through the intersection the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.
23. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15, respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

SECTION - D

24. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by (a, b) R (c, d). If $a + d = b + c$ for (a, b), (c, d) in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].

OR

Prove that the relation R in set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

25. Find the equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \leq x \leq 2\pi$, that are parallel to the line $x + 2y = 0$.

OR

Find the radius of the smallest circular with centre on Y-axis and passing through the point (7, 3).

26. Two groups of students representing 'SEVEN MOTHER EARTH' and 'GO GREEN' are standing on two planes represented by the equations.

$$\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5 \text{ and } \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 8.$$

What is the angle between the planes? Name few activities which should be taken up to save mother earth.

27. Find the area of the region bounded by a circle $4x^2 + 4y^2 = 9$ and the parabola $y^2 = 4x$.

28. Solve the following initial value problem. $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0, y(1) = 2$

OR

Solve the following differential equation. $(1 + y + x^2y)dx + (x + x^3)dy = 0$.

29. Let a, b and c denote the sides BC, CA and AB respectively of ΔABC . If $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then

find the value of $\sin^2 A + \sin^2 B + \sin^2 C$.