MATHEMATICS **PAPER-6**

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

2. Find a vector in the direction of vector $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, which has magnitude 8 units.

3. Let $f: R \to R : f(x) = (x^2 - 3x + 2)$. Find $fof(x)$.

4. Let $A = \{0,1,2,3\}$ and define a relation R or A as follows $R = \{(0,0)(0,1)(0,3)(1,0)(1,1)(2,2)(3,0)(3,3)\}$. Is R reflexive, symmetric and transitive?

SECTION - B

5. Show that the points $(a+5, a-4), (a-2, a+3)$ and (a, a) do not lie on a straight line for any value of a.

6. If $P(A) = \frac{1}{4} P(B) = \frac{1}{5} and P(A \cap B) = \frac{1}{7}$, find $P(\overline{A} / \overline{B})$.

7. Determine $f(0)$, so that the function $f(x)$ define by $f(x) = \frac{(4^x - 1)^3}{\sin\frac{x}{4} \log\left(1 + \frac{x^2}{3}\right)}$.

8. If x changes from 3 to 3.3, find the approximate change in $\log_e(1+x)$.

9. If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y=\tan^{2}\frac{x}{2}$.

10. If $y=b\tan^{-1}\left(\frac{x}{a}+\tan^{-1}\frac{y}{x}\right)$, *find* $\frac{dy}{dx}$.

11. Evaluate $\int \tan(x-\theta)\tan(x+\theta)\tan 2x\,dx.$

12. Find the position vector of a point R which divides the line joining the points $p\left(\hat{i}+2\hat{j}-\hat{k}\right)$ *and* $Q\left(-\hat{i}+\hat{j}+\hat{k}\right)$ in the ratio 2:1

(i) internally. (ii) externally

SECTION - C

13. Find the shortest distance between lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

14. Find the particular solution of the differential equation $\left(1+e^{2x}\right)dy+\left(1+y^{2}\right)e^{x}dy=0,$ given that $y=1$ when x=0.

15. Show that the differential equation that represents the family of all parabolas having their axis of symmetry coincident with the axis of x is $yy_2+y_1^2=0$.

16. Evaluate $\int_0^1 e^{3x-2}dx$.

17. Evaluate $\int\sqrt{3-4x-4x^2}\,dx$.

OR

Evaluate $\int\frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$.

18. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and postel –sheets made by them using recycled paper, at the rate of `20, `15 and `5 per unit, respectively. School A sold 25 paper bags, 12 scrap-books and 34 postel-sheets. School B sold 22 paper bags, 15 scrap-books and 28 postel-sheets, while school C sold 26 paper bags, 18 scrap-books and 36 postel-sheets. Using matrices, find the total amount raised by each school. By such exhibition, which values are inculcated in the students?

19. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

OR

If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$, show that $\left(x^2 + a^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$

20. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 17$ is increasing or decreasing.

21. Prove that $a.(b+c)\times(a+2b+3c) = [abc]$.

22. Find the mean and variance of number of tails when a coin is tossed thrice.

OR

12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box.
If it is known that the number on the drawn card is more than 3, then find the probability that it is an even number.

23. Three bags contain a number of red and white balls as follows
Bag I: 3 red balls, Bg II: 2 red balls and 1 white ball and Bag III: 3 white balls. The probability that bag will be chosen and a ball is selected from it is $\frac{i}{6}$, where $i = 1, 2, 3$.

(i) What is the probability that a red ball is selected?

(ii) If a white ball is selected, then, what is the probability that it came from bag III?

SECTION - D

24. Find the area of the region bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$.

25. Let T be the set of all triangles in a plane. Let us define a relation.
$R=\{(T_1, T_2): T_1 \text{ is similar to } T_2;\ T_1, T_2 \in T\}$.
Show that r is an equivalence relation.

26. Form the curve $y = 4x^3 - 2x^5$, find all the points on the curve at which the tangent passes through the origin.

OR

Show that of all the rectangles with a given perimeter, the square has the largest area.

27. Find the image of point (1, 0, 0) on the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

OR

Find the equation of the plane that contains the point (1, −1, 2) and is perpendicular to both the planes $2x+3y-2z=5$ and $x+2y-3z=8$. Hence, find the distance of point $P(-2,5,5)$ from the plane obtained above.

28. An aeroplane can carry a maximum of 200 passengers. A profit of `1000 is made on each executive class ticket and a profit of `600 is made on each economy class ticket.

The airline donate its 5% of total profit in welfare fund for poor girls. The airline reserves atleast 20 seats for executive class. However, atleast 4 times as many passengers prefer to travel by economy class, then by executive class. Determine how many tickets of each type must be sold in order to maximise profit for the airline? What is the maximum profit?
Do you think, more passengers would prefer to travel by such an airline than by others?

29. Show that ΔABC is an isosceles triangle, if the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix} = 0.$$

OR

If $A+B+C=\pi$, show that

$$\begin{vmatrix} \sin^2 A & \sin A\cos A & \cos^2 A \\ \sin^2 B & \sin B\cos B & \cos^2 B \\ \sin^2 C & \sin C\cos C & \cos^2 C \end{vmatrix} = -\sin(A-B)\sin(B-C)\sin(C-A).$$