MATHEMATICS PAPER-5

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. Find λ so that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar.

2. Evaluate $\int_1^2 \log_e [x] dx,$ where [.] denotes the greatest integer function.

3. Find $\int \frac{8^{1+x} + 4^{1-x}}{2^x} dx$.

4. Find the values of a and b, if $A = B$, where $A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} and\ B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$.

SECTION - B

5. If A is a skew-symmetric matrix of odd order n, then show that $|A| = 0$.

6. Using differentials, find the approximate value of $(82)^{1/4}$ up to three places of decimal.

7. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}\ and\ 3\hat{i} - 2\hat{j} + \hat{k}$.

8. An airline agrees to charter planes for a group. The group needs atleast 160 first class seats and atleast 300 tourist class seats. The airline must use atleast two of its model 314 planes which have 20 first class and 30 tourist class seats. The airline will also use some of its model 535 planes which have 20 first class seats and 60 tourust class seats. Each flight of a model 314 plane costs the company `100000 and each flight of a model 535 plane costs `150000. How many of each type of plane should be used to minimize the flight cost? Formulate this as a LPP.

9. The random variable X can take only the values 0, 1, 2. Given that P(X=0)=P(X=1)=p and that $E(X^2)=E(X)$, find the value of p.

10. Find the local maxima and local minima of the function $f(x)=(\sin x-\cos x)$, where $0<x<2\pi$.

11. Evaluate $\int \{1+2\tan x(\tan x+\sec x)\}^{1/2} dx$.

12. Find the value of k for which $f(x)=\begin{cases} kx+5, & when\, x\le 2 \\ x-1, & when\, x>2 \end{cases}$ is continuous at x=2.

SECTION - C

13. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r}=\left(-3\hat{i}+\hat{j}+5\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}-5\hat{k}\right)$.

14. Find the minimum value of $\left(\sec^{-1}x\right)^2+\left(\operatorname{cosec}^{-1}x\right)^2$.

15. If $\hat{a}\, and\, \hat{b}$ are unit vectors inclined at an angle θ, then prove that $\sin\frac{\theta}{2}=\frac{1}{2}\left|\hat{a}-\hat{b}\right|$.

16. A librarian has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weight 1 kg and $1\frac{1}{2}kg$ each, respectively. The shelf is 96 cm long and atmost support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically.

17. In Arjun's school, annual sports meet with the title 'Sports for Healthy Life' was being organised. On the last day of the meet, there was the event of hurdle race. In this hurdle, it was decided that a player has to cross 10 hurdles.
From the past games experience it can be said that the probability of a player to clear each hurdle will be 5/6. Find the probability that a player will knock down fewer than 2 hurdles.

OR

A couple has 2 children. Find the probability that both are boys, if it is known that
(i) one of them is a boy.
(ii) The older child is a boy

18. If $x=\sqrt{a^{\sin^{-1}t}}\, and\, y=\sqrt{a^{\cos^{-1}t}}, a>0\, and\, -1<t<1$, then prove that $\frac{dy}{dx}=\frac{-y}{x}$.

OR

Ina given function $f(x)=x^3+bx^2+ax,\ x\in[1,3]$, Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$. Find the values of a and b.

19. Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y'}$ given that $y=\frac{\pi}{2}, when\, x=1$.

20. Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of `1100 for the three values of 5, 4 and 3 students, respectively, while school B decided to award `10700 for the three values of 4, 3 and 5 students, respectively. If all the three prizes together amount to `2700, then

(i) Represent the above situation by a matrix equation and form linear equations using and form linear equations using matrix multiplication.

(ii) Is it possible to solve the system of equations, so obtained using matrices?

(iii) Which value you prefer to be rewarded most and why?

21. Evaluate $\int \frac{\sin x+\cos x}{9+16\sin 2x}dx.$

OR

Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)}dx.$

22. Prove that $\int_0^{2\pi} \frac{x\sin^{2n} x}{\sin^{2n} x+\cos^{2n} x}dx=\pi^2$.

23. Bag a contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One balls is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

SECTION - D

24. Find the equation of tangent to the curve given by $x=\alpha\sin^3 t\, and\, y=b\cos^3 t$ at a point $t=\pi/2.$

OR

A telephone company in a town has 500 subscribers on its list and collects fixed charge of `300 per subscriber per year. The company proposes to increase the annually subscription and it is believed that for every increase of `1, one subscriber will discontinue the service. Find what increase will bring maximum profit?

25. Find the area of the region bounded by the curves $y^2=4ax\, and\, x^2=4ay.$

OR

Using integration, find the area of the triangular region whose sides have the equation $y=2x+1, y=3x+1\, and\;\, x=4.$

26. Considering the Earth as a plane having equation $\vec{r}\left(2\hat{i}+2\hat{k}\right)=8$ and the Mars as a line having equation $\vec{r}=\hat{i}-2\hat{j}+\hat{k}+\lambda\left(2\hat{i}+3\hat{j}+\hat{k}\right)$ such that their common point is (α,β,γ). A monument is standing vertically on the Earth such that its peak is at the point $(-2,-6,-11)$.

(i) Find the distance between the peak and common point of Earth and Mars.

(ii) How can we save our monuments?

27. Let $f,g:\mathrm{R}\rightarrow\mathrm{R}$ be two functions defined as $f(x)=|x|+x, g(x)=|x|-x, \forall x\in\mathrm{R}$. Then, find fog and gof.

28. Solve the given differential equation $\sqrt{1-y^2}\,dx=\left(\sin^{-1}y-x\right)dy, y(0)=0.$

29. Using properties of determinants, prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}=1+a^2+b^2+c^2$

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