MATHEMATICS PAPER-4

General Instruction:

1. *All questions are compulsory.*
2. *This question paper contains 29 questions.*
3. *Question 1-4 in Section A are very short answer type questions carrying 1 mar each.*
4. *Questions 5-12 in Section B are short answer type questions carrying 2 marks each.*
5. *Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.*
6. *Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.*
7. *There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.*

Time- 3 Hours *[Max. Marks-100]*

SECTION -A

1. Show by an example that for $A \neq O$ and $B \neq O$, $AB = O$.

2. Differentiate $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$ w.r.t. x.

3. Find the value of λ, so that the vectors $\vec{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ *and* $\vec{b} = \hat{i}+\lambda\hat{j}+3\hat{k}$ are perpendicular to each other.

4. If $x^{2/3}+y^{2/3}=a^{2/3}$, then find $\frac{dy}{dx}$.

SECTION - B

5. Prove that $\vec{a}.\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+2\vec{b}+3\vec{c}\right)=\left[\vec{a}\vec{b}\vec{c}\right]$.

6. If $I_n = \int_0^{\pi/4} \tan^n x dx$, prove that $I_n + I_{n+2} = \frac{1}{n+1}$.

7. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ *and* $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$.

8. Find the integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$.

9. A resourceful home decorator manufactures two types of lamps say A and B. Both lamps go through two technicians, first a cutter, second a finisher. Lamp A requires 2 h of the cutter's time and 1 h of the finisher's time. Lamp B requires 1 h of cutter's and 2 h of finisher's time.

The cutter has 104 h and finisher has 76h of time available month. Profit on one lamp A is `6.00 and on one lamp B is `11.00. Assuming that he can sell all that he produces, how many of each type of lamps should he manufacture to obtain the best return.

10. Suppose $f(x) = ax^2 + bx^2 + cx + d, x \in [0,1]$ is continuous in given closed interval and differential in open interval. Then, verify the Lagrange's mean value theorem.

11. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing.

12. One card is drawn at random from a pack of well –shuffled deck of cards.
Let E: the card drawn is a spade
F: the card drawn is an ace
Are the events E and F independent?

SECTION - C

13. If a young man rides his motorcycle at 25km/h, then he has to spend `2 per km on petrol. If he rides at a faster speed of 40km/h, then the petrol cost increases at `5 per km.

He has `100 to spend on petrol and wishes to find what is the maximum distance, he can travels within one hour. Expresses this as a linear programming problem and solve it graphically.

14. Using differentials, find the approximate value of $(0.999)^{1/10}$.

15. If $y = (\cos x)^{(\cos x)^{(\cos x)\ldots\infty}}$, show that $\dfrac{dy}{dx} = \dfrac{y^2 \tan x}{y \log \cos x - 1}$.

16. Prove that following.

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2}; x \in \left(0, \frac{\pi}{4}\right).$$

17. Prove that $\int_0^1 \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) dx = \dfrac{\pi}{2} - \log 2$.

OR

Evaluate $\int_0^2 \left[x^2\right] dx$, where [.] is the greatest integer function.

18. Evaluate $\int \sqrt[3]{\dfrac{\sin^2 x}{\cos^{14} x}} dx.$

19. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

OR

If $\vec{a}$ *and* $\vec{b}$ are two non-zero vectors, then prove that $\left(\vec{a}\times\vec{b}\right)^2 = \begin{bmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{bmatrix}$.

20. Find the equation of the plane passing through the point $(1,1,-1)$ and perpendicular to the planes $x+2y+3z-7=0 \; and \; 2x-3y+4z=0$.

21. Two dice are thrown simultaneously. Let x denotes the number of sixes. Find the probability distribution of X.

OR

A and B are two candidates seeking admission in a college. The probability that A is selected, is 0.7 and the probability that exactly one of them is selected, is 0.6. Find the probability that B is selected.

22. To promote the making of toilets for women, an organization tried to generate awareness through

(i) house calls (ii) letters and (iii) announcements

(i) ₹50 (ii) ₹20 (iii) ₹40

The number of attempts made in three villages X, Y and Z are given below

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately, using matrices. Write one values generated by the organization in the society.

23. By examining the MRI result, the probability that cancer is detected when a person is actually suffering, is 0.99. The probability of a healthy person diagnosed to have cancer is 0.001. In a certain city, 1 in 1000 people suffers from cancer. A person is selected at random and is diagnosed to have cancer. What is the probability that he actually has cancer?

SECTION - D

24. Let X be a non –empty set and P(X) be its power set. Let '*' be an operation defined on elements of P(X) by

$A*B = A\cap B, \forall A,B \in P(X)$. Then,

(i) Prove that '*' is a binary operation in P(X)
(ii) Prove that '*' is commutative.
(iii) Prove that '*' is associative.
(iv) If 'O' is another binary operation defined on P(X) as $AOB = A\cup B$, then verify that 'O' distributes over '*'.

OR

Find f^{-1}, if $f: R\to(-1,1)$ is defined by $f(x) = \dfrac{\sqrt{7}^{x}-\sqrt{7}^{-x}}{\sqrt{7}^{x}+\sqrt{7}^{-x}}$.

25. Using the method of integration, find the area bounded by the curve $|x|+|y|=1$.

26. Prove that the surface area of a solid cuboid of square base and given volume is minimum when it is a cube.

OR

If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $(\pi / 3)$.

27. Find the equation of the plane containing the lines $\vec{r} = \hat{i} + \hat{j} + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \hat{i} + \hat{j} + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$.

Find the distance of this plane from the origin and also from the point (2, 2, 2).

28. Solve the following system of equations by matrix method when $x \neq 0, y \neq 0 \, and \, z \neq 0$.

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$ and $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

OR

The sum of three numbers is 6. Twice the third number when added to the first number gives 7. On adding the sum of the second and third numbers to thrice the first number, we get 12. Find the numbers, using matrix method.

29. $(x-1)\,dy + y\,dx = x(x-1)\,y^{1/3}dx$, where x denotes the percentage of population living in a city and y denotes the area for living a healthy life of population. Find the particular solution, when x=2 and y=1. Is higher density of population is harmful? Justify your answer.

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