MATHEMATICS **PAPER-3**

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is no overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. If the percentage error in the edge of a cube is 1, then find error in its volume.

2. If A is a matrix of order 2×3 and B is a matrix of order 3×5, then what is the order of matrix $(AB)'or\ (AB)^T$?

3. Evaluate $\int_{-2}^{1}\frac{|x|}{x}dx$.

4. Write the direction cosines of vector $2\hat{i}-\hat{j}+3\hat{k}$.

SECTION - B

5. If $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$, find A^2-4A+I.

6. Let A and B be two events of the same sample space S of an experiment, then prove that $0\le P(A/B)\le1, B\ne\phi$.

7. Differentiate $\sin^2(3x+1)$ w.r.t.x.

8. Find the point on the curve $y^2=8x$ for which the abscissa and ordinate change at the same rate.

9. Find the slope of the tangent and normal to the curve $y=(\sin2x+\cot x+2)^2$ $at\ x=\frac{\pi}{2}$.

10. Evaluate $\int(x+1)e^x\log(xe^x)dx$.

11. Use differentials, find the approximate value of $(0.037)^{1/2}$.

12. If the vector $\vec{\alpha} = a\hat{i} + \hat{j} + \hat{k}, \vec{\beta} = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{\gamma} = \hat{i} + \hat{j} + c\hat{k}$ **are coplanar, then prove that** $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$, where $a \neq 1,\ b \neq 1 and\ c \neq 1$.

SECTION - C

13. Evaluate $\int \frac{dx}{1-3\sin x}$

OR

Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$.

14. If $y = a(1+\cos\theta) and\ x = a(\theta - \sin\theta)$, find $\frac{d^2y}{dx^2} at\ \theta = \frac{\pi}{2}$.

OR

If $\cos\frac{x}{2}.\cos\frac{x}{4}.\cos\frac{x}{8}.... = \frac{\sin x}{x}$, prove that $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{4} + ... = \cos ec^2 x - \frac{1}{x^2}$.

15. Find the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$, if the slope of the tangent to the curve at any point $P(x,y)$ is $\frac{y}{x} - \cos^2\frac{y}{x}$.

16. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$ is increasing in R.

17. Mr. X has invested a part of his investment in 10% bond A and a part in 15% bond B. His interest income during first year is `4000. If he invests 20% more in 10% bond A and 10% more in 15% bond B, his income during second year increases by `500. Find the initial amount of investment in respective bonds, using matrix method. Is investment necessary a middle class?

18. Solve the differential equation $(x-y)(dx+dy) = dx - dy, y(0) = -1$.

19. If $y = \cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y = \tan^2\frac{x}{2}$.

20. A speaks the truth 8 times out of 10 times. He tossed a die. He reports that it was 5. What is the probability that it was actually 5?

OR

A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. If there is atleast one girl on the committee, then calculate the probability that there are exactly 2 girls on the committee.

21. Using vectors, find the area of triangle with vertices A(2, 3, 5), B(3, 5, 8) and C(2, 7, 8).

22. Find the angle between the lines $\frac{2-x}{-5}=\frac{3+y}{3}=\frac{z}{2}$ *and* $\frac{x+2}{-1}=\frac{3y-5}{2}=\frac{z-5}{4}$.

23. A coin is biased such that a heat a is three times as likely to occur than a tail. When it is tossed twice, then find the probability distribution of number of heads. Also, find then mean and variance.

SECTION - D

24. Let $A=R-\{3\}$, $B=R-\{1\}$. If $f:A\to B$ be defined by $f(x)=\frac{x-2}{x-3}, \forall x\in A$. Show that f is bijective and find the inverse of f.

25. Find the angle of intersection of the curves $y^2=x$ *and* $x^2=y$.

OR

A metal box with a square base and vertical sides is to contain 1024 cm^3. If the material for the top and bottom costs `5 per $cm^2$ and the material for the sides costs `2.50 per cm^2. Then, find the least cost of the box.

26. Find the area bounded by the curve $y=\cos x$ between $x=0$ and $x=2\pi$.

OR

Find the area bounded by the curve $y^2=4a^2(x-1)$ and the lines $x=1$ and $y=4a$.

27. If $a=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ *and* $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ find AB. Use this to solve the system of equations $x-y=3, 2x+3y+4z=17$ and $y+2z=7$.

OR

By using elementary row operations, find the inverse of the matrix $A=\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{bmatrix}$.

28. Two ships in the sea were reported missing their link with ground control suddenly. They were on the lines

$\vec{r}=(2+\lambda)\hat{i}-(3+\lambda)\hat{j}+(5+\lambda)\hat{k}$ and $\vec{r}=(2\mu-1)\hat{i}+(4\mu-1)\hat{j}+(5-3\mu)\hat{k}$.

Using shortest distance formula, determine whether they met any mishappening. What value do you see in it.

29. A dealer wishes to purchase a number of fans and radios. He has only `5760 to invest and has a space for atmost 20 items. A fan costs him `360 and a radio ` 240. His expectation is that he can sell a fan at a profit of `22 and a radio at a profit of `18. Assuming that, he can sell all the

items that he buys, how should he invest his money for maximum profit? Translate the problem as LPP and solve it graphically.

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