General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.

(v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.

(vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

SECTION A

1. Check whether the solution R on R defined as $R=(a,b):(a\ge b^3)$ is reflexive.

2. If A is an invertible matrix of order 3 × 3 and $|A| = 9$, then find $ad(adj\ A)$.

OR

For what value of x, is the matrix $A=\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew symmetric matrix?

3. Write the value of cosine of the angle which the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ makes with y-axis.

4. Let * be a binary operation on N defined by $a*b=a^b$, find the value $f(3*2)*1$.

SECTION B

5. Find the general solution of the differential equation

$$\frac{dy}{dx}=(1+x^2)(1+y^2)$$

6. If $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$ what can you conclude about $\vec{a}$ and $\vec{b}$?

7. Events E and F are independent. Find $P(F)$, if $P(E)=\frac{2}{5}$ and $P(E\cup F)=\frac{3}{5}$

OR

IF E and F be two events such that $P(E)=\frac{1}{3}$, $P(F)=\frac{1}{4}$, find $P(E\cup F)$ if E and F are independent events.

8. Evaluate : $\int_0^2 (x-3)dx$, if $f(x)=\begin{cases} -(x-3), x<2 \\ +(x-3), x>2 \end{cases}$

9. Prove that $\cos^{-1}x=2\sin^{-1}\sqrt{\frac{1-x}{2}}$

OR

Write in the simplest form : $\sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

10. Discuss the commutativity of the binary operation * on R defined by $a*b=a-b+ab$ for all $a,b\in R$, .

11. If $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right), x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ find $\frac{dy}{dx}$.

12. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then for what value of α when A is an identity matrix ?

OR

Find the values of x and y from the following matrix equation $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$

SECTION C

13. Show that : $f(x):\begin{cases}3x-2 & \text{if} \quad 0<x<1\\ 2x^2-x & \text{if} \quad 1<x<2\\ 5x-4 & \text{if} \quad x>2\end{cases}$ is continous at $x = 2$

14. A rectangle is inscribed in a semicircle of radius r with one of its side on the diameter of the semi-circle. Find the maximum area of the rectangle.

15. If $\sin y=x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

OR

Verify Rolle's theorem for the function $f(x)=x^3-6x^2+11x-6$ on the interval $[1,3]$.

16. Show that the normal at any point θ to the curve $x=\alpha\cos\theta+\alpha\theta\sin\theta$ and $y=a\sin\theta-a\theta\cos\theta$ is at a constant distance form origin.

OR

Find the intervals in which the function f given by $f(x)=\sin x+\cos x, 0\le x\le 2\pi,$ is strictly increasing or strictly decreasing.

17. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ *and* $(A+B)^2=A^2+B^2$, find a and b

18. Evaluate : $\int x^2\cot^{-1}x\,dx$

19. If $\vec{a}=\hat{i}+2\hat{j}+\hat{k}, \vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{i}-4\hat{j}-5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a}-\vec{b})$ and $(\vec{c}-\vec{b})$.

20. Solve the following differential equation :

$$x^2\frac{dy}{dx}=y^2+2xy$$

Given that $y=1$, when $x=1$

OR

Solve the differential equation

$$\left(1+e^{2x}\right)dy+\left(1+y^2\right)e^x dx=0$$

21. Find the direction cosines of the vector joining the points $A(1,2,-3)$ and $(-1,-2,1)$, directed from A to B.
22. In a class having 60% boys, 5% of the boys and 10% of the girls have an I.Q. of more than 150. A student is selected at random and found to have an I.Q. of more than 150. Find the probability that the selected student is a boy.
23. Given that $P(\text{not } F)=0.65$ and $P(E\cup F)=0.85$ where, E and F are independent events. Find P(E).

SECTION D

24. Show that the function $F:R\to\{x\in R:-1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}, x\in R,$ is one-one and onto.
25. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

OR

Using properties of determinants prove the following:

$$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

26. Using integration, find the area of the region enclosed between the circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$.
27. Show that the lines

$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect each other.

OR

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, with its vertex at one end of the major axis.

28. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of types B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs.5 each for type A and Rs.6 for type B

souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

29. Evaluate : $\int \frac{1}{3+2\cos x}\,dx$

OR

Evaluate : $\int \frac{dx}{2-\sin x}$