General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.

(v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.

(vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

SECTION A

1. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}, xy < 1$, then write the value of $x + y + xy$.

2. Write the number of vectors of unit length perpendicular to both the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$

OR

Write the sum of intercepts cut off by the plane $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) - 2 = 0$ on the three axes.

3. Let A and B are matrices of order 3×2 and 2×4 respectively. Write the order of matrix (AB).

4. If $f : R \to R$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function, then find $f^{-1}(x)$

SECTION B

5. If $y = \tan^{-1} \frac{5x}{1-6x^2}, -\frac{1}{\sqrt{6}} < x, \frac{1}{\sqrt{6}}$. Then prove that $\frac{dy}{dx} = \frac{2}{1+4x^2} + \frac{3}{1+9x^2}$

6. Prove that the diagonal elements of a skew symmetric matrix are all zero.

7. Verify that $ax^2 + by^2 = 1$ is a solution of differential equation $x\left(yy_2 + y_1^2\right) = yy_1$.

OR

Obtain the differential equation of the family of circles having centre $(0, b)$ or centre lying on y-axis passing through the points $(a, 0)$ and $(-a, 0)$

8. The radius r of a right circular cylinder is increasing uniformly at the rate of 0.3 cm/s and its height h is decreasing a the rate of 0.4 cm/s. When r = 3.5 cm and h = 7 cm, find the rate of change of the curved surface area of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

9. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all? Justify your answer.

OR

Let $A=(a,b,c)$ and $B=(1,2,3)$. Find f^{-1} of the following function *f* from *A* to *B*, if it exists.

(i) $f=\{(a,3)(b,2)(c,1)\}$

(ii) $f=\{(a,2)(b,1)(c,1)\}$

10. Find the Cartesian equation of the line which passes through the point $(-2,4,-5)$ and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$

11. A couple has 2 children. Find the probability that both are boys, if it is known that
(i) one of them is a boy (ii) the older child is a boy

12. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs.4 km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs.5/ km. She has Rs.200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem.

OR

A firm has to transport atleast 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs.400 and each small van is Rs.200. Not more than Rs.3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

SECTION C

13. Find the particular solution of the differential equation $2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0,$ given that $x=0$ when $y=1$.

OR

Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}=\left(e^{m\tan^{-1}x}-y\right)$ given that $y=1$ when $x=0$.

14. A farmer has a field of shape bounded by $x=y^2$ and $x=3$, he wants ot divide this into his two sons equally by a straight line $x=c$. Can you find c?

15. Evaluate : $\int_1^3 (x^2+x)\,dx$

16. Find the intervals in which the function $f(x)=(x-1)^3(x-2)^2$ is
(i) increasing (ii) decreasing

OR

Find the equation of tangent to curves.

$x = \sin 3t, y = \cos 2t$ at $t = \dfrac{\pi}{4}$

17. If $x = a\cos\theta = b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then prove that : $y^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$

18. Using properties of determinants, prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

OR

Using properties of determinants prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

19. Find the equation of the plane passing through the line of intersection of the plane $\vec{r}\cdot\left(\hat{i} + 3\hat{j}\right) - 6 = 0$ and $\vec{r}\cdot\left(3\hat{i} - \hat{j} - 4\hat{k}\right) = 0$, which is at a unit distance from the origin.

20. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, prove that the vectors $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are also coplanar.

21. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

22. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining third six in the sixth throw of die.

23. A man has Rs.1500 for purchasing wheat and rice. A dbag of rice and a bag of wheat cost Rs.180 and Rs.120 respectively. He has a storage capacity of only 10 bags. He earns a profit of Rs.11 and Rs.9 per bag of rice and wheat respectively. Formulate the problem as an LPP to find the number of bags of each type he should buy for getting maximum profit and solve it graphically.

SECTION D

24. Find : $\displaystyle\int \frac{x^2 dx}{\left(x\sin x + \cos x^2\right)} dx$

OR

Find : $\int \frac{\sqrt{x^2+1}\left\{\log\left(x^2+1\right)-2\log x\right\}}{x^4}dx$

25. If Z is the set of all integers and R is the relation on Z defined as $R=\left\{(a,b):a,b\in Z \text{ and } a-b \text{ is divisible by } 5\right\}$ Prove that R is an equivalence relation.

OR

Does the following trigonometric equation have any solutions ? If yes, obtain the solution(s):

$$\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=-\tan^{-1}7$$

26. If $x=\cos\theta$ and $y=\sin^3\theta$ then prove that:

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)=3\sin^2\theta\left(5\cos^2\theta-1\right)$$

OR

Does the following trigonometric equation have any solutions ? If yes, obtain the solution(s);

$$\tan^{-1}\left(\frac{x-1}{x-1}\right)=\tan^{-1}\left(\frac{x-1}{x}\right)=-\tan^{-1}7$$

27. Find the point on the curve $y^2=2x$ which is at a minimum distance from the point (1, 4)

28. A mixture is to be made of three foods A, B, C. The three foods A, B, C contains nutrients P, Q, R as shown below:

Food	**Gram per keg of nutrient**		
	P	**Q**	**R**
A	1	2	3
B	3	1	1
C	4	2	1

How to form a mixture which will have 8 gram of P, 5 gram of Q and 7 gram of R?

29. Find the equation of the plane passing through the point (3, –3, 1) and perpendicular to the line joining the points (3, 4, –1) and (2, –1, 5). Also find the co-ordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane.