General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.

(v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.

(vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

SECTION A

1. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A-I)^3 + (A+I)^3 - 7A$

2. Write the distance between the parallel planes $2x - y + 3z = 4$ and $2x - y + 3z = 18$

3. Evaluate : $\int \cos^{-1}(\sin x)\,dx$

4. Examine that $\sin|x|$ is a continuous function.

OR

Write the statement of mean value theorem.

SECTION B

5. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$, show that $AB \neq BA$

6. If $e^y(x+1) = 1$, show that $\frac{dy}{dx} = -e^5$

7. The radius r of the base of a right circular cone is decreasing at the rate of 2 cm/min and height h is increasing at the rate of 3 cm/min. When $r = 3.5$ cm and $h = 6$cm , find the rate of change of the volume of the come. $\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

Find the value of *a* if tangent to curve $y = x^2 - ax + 7$ is parallel to the line $2x - y + 9 = 0$ at $(-1,1)$

8. Discuss the differentiability of the function.

$$f(x) = \begin{cases} wx - 1, & x < \frac{1}{2} \\ 3 - 6x, & x \geq \frac{1}{2} \end{cases} \text{ at } x = \frac{1}{2}$$

9. If either vector $\vec{a} = 0$ or $\vec{b} = 0$ then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer.

10. Show that the solution of differential equation

$$y = 2\left(x^2 - 1\right) + ce^{-x^2} \text{ is } \frac{dy}{dx} + 2xy - 4x^3 = 0$$

OR

Find the general solution of differential equation $\frac{dy}{dx} = e^{3x-4y}$

11. Prove that $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$ if $x^2 - y^2 \leq 1$

12. Find $\int \frac{a}{b - ce^x} dx$

OR

If $\int_0^1 \left(3x^2 + 2x + k\right) dx = 0,$ then find the value of k.

SECTION C

13. Find the position vector of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio 1:2. Also, show that P is the mid-point of the line segment PQ.

14. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ and each one of them is perpendicular to the sum of the other two, then find $|\vec{a} + \vec{b} + \vec{c}|$

15. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)\left(a^2+b^2+c^2\right)$$

16. If $y = \sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$ and $0 < x < 1,$ then find $\frac{dy}{dx}$

OR

Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right),$ when $x \neq 0$.

17. Find : $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx, \ x \in [0,1]$

OR

Find : $\int \frac{1}{\cos^4 x + \sin^4 x} dx$

18. Evaluate : $\int_0^1 \frac{\log|1+x|}{1+x^2} dx$

19. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, then prove that $\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos a + \frac{y^2}{b^2}$

20. From the differential equation representing family of ellipses having foci on x-axis and centre at the origin

21. Solve the differential equation given as :

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} + \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1, x \neq 0$$

22. If * is binary operation on Q, defined by $a*b = \frac{3ab}{5}$. Show that * is commutative as well as associative. Also, find its identity, if it exists.

OR

Show that the relation R in the set $N \times N$ defined by $(a,b)R(c,d)$ if $a^2 + d^2 = b^2 + c^2 \ \forall a,b,c,d \in N,$ is an equivalence relation.

23. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%.

SECTION D

24. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$, find $(AB)^{-1}$

OR

If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 3 & -20 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2; \frac{4}{x} + \frac{6}{y} + \frac{5}{z} = 5; \ \frac{6}{x} + \frac{9}{y} + \frac{20}{z} = 4$

25. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

OR

Prove that the height of the cylinder or maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

26. Evaluate: $\int_1^3 \left(2x^2+5x\right)dx$ as a limit of sums.

OR

Find the area of region $\left\{(x,y);x^2 \le y \le |x|\right\}$

27. Find the co-ordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point $P(5,4,2)$ to the line $\vec{r}=-\hat{i}+3\hat{j}+\hat{k}+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$. Also find the image of P in this line.

28. In a class, 5% of boys and 10% of girls have an IQ of more than 150. In the class, 60% are boys and rest are girls. If a student is selected at random and found to have an IQ of more than 150, then find the probability that the student is a boy.

29. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. food I contains 2 units/kg of vitamin A and I unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs.5 per kg to purchase food I and Rs.7 per kg to purchase food II. Determine the minimum cost for such a mixture. Formulate the above as a LPP and solve it graphically.

LETS PLAY WITH MATHS