General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.

(v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.

(vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

SECTION A

1. Find the value of p, if $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+3\hat{j}+p\hat{k}\right)=\vec{0}$

2. Write the value of $x+y+z$ if $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$

3. Find the derivative of $f\left(e^{\tan x}\right)$ w.r.t. x at $x=0$. It is given that $f'(1)=5$

OR

Using derivative, find the approximate percentage increase in the area of a circle if its radius is increased by 2%

4. Evaluate: $\int\frac{(1+\log x)^2}{x}dx$

SECTION B

5. Find the point on the curve $y^2=8x+3$ for which the y coordinate changes 8 times more than coordinate of x.

6. If $A=[1\ 2\ 3]$ and $B=\begin{bmatrix}-2\\3\\1\end{bmatrix}$, find AB and BA.

7. If $y=\tan^{-1}\sqrt{\frac{1-x}{1+x}}$, find $\frac{dy}{dx}$

8. If $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$, then show that $x=\frac{a+b}{1-ab}$

OR

Write the value of the following:

$$\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$$

9. Find $\int e^x\left[\sec x+\log\left|\sec x+\tan x\right|\right]dx$

10. Find the distance between the planes $2x+3y+4z=10$ and $4x+6y+8z=18$

OR

Find the scalar components of the vector $\overline{AB}$ with initial point A(2,1) and terminal point B (–5, 7)

11. Show that the tangents to be curve $y=x^2-7x+18$ at $(3,0)$ and $(4,0)$ are at right angles.

12. If A and B are two events such that $P(A)=0.4, P(B)=0.8$ and $P(B/A)=0.6,$ then find $P(A/B)$.

OR

Three cards are drawn without replacement from a pack of 52 cards. Find the probability that

(i) the cards drawn are king, queen and jack respectively.

(ii) the cards drawn are king, queen and jack.

SECTION C

13. Prove the following: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}=4a^2b^2c^2$

OR

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}=1+a^2+b^2+c^2$$

14. If $y=x\log\left(\frac{x}{a+bx}\right)$, then prove that : $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$

OR

Let $y=(\log x)^x+x^{x\cos x}$, then find $\frac{dy}{dx}$

15. Evaluate : $\int\frac{x^2+1}{(x-1)^2(x+3)}dx$

OR

Evaluate : $\int\frac{x^2+1}{x^4+1}dx$

16. Show that: $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}=\tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}$

17. Evaluate: $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

18. Form the differential equation of the family of circles in the second quadrant and touching the co-ordinate axes.

19. Let $f:N\to N$ be defined as $f\left(n\right)=\begin{cases}\dfrac{n+1}{2}, \text{ when } n \text{ is odd number}\\ \dfrac{n}{2}, \text{ when } n \text{ is even number}\end{cases}$

for all $n\in N$. State whether the function f is bijective. Justify your answer.

20. If the vector $\vec{p}=a\hat{i}+\hat{j}+\hat{k},\ \vec{q}=\hat{i}+b\hat{j}+\hat{k}$ and $\vec{r}=\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then for $a,b,c\neq1$, then show that

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

21. Find the shortest distance between the following pairs of skew lines:

$$\frac{x-1}{2}=\frac{2-y}{3}=\frac{z+1}{4}$$

$$\frac{x+2}{-1}=\frac{y-3}{2}=\frac{z}{3}$$

22. A problem in mathematics is given to 4 students A, B, C and D. Their chances of solving the problem respectively are $\frac{1}{3},\frac{1}{4},\frac{1}{5}$ and $\frac{2}{3}$. What is the probabiolity that (i) the problem will be solved? (ii) at most one of them solve the problem?

23. A girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she threw 1, 2, 3 or 4 with the die?

SECTION D

24. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$

OR

Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

25. Prove that the curves $y^2=4x$ and $x^2=4y$ divide the area of square bounded by $x=0, x=4, y=4$ and $y=0$ into three equal parts.

OR

Using integration, find the area of the region bounded by the two parabolas $y^2=4x$ and $x^2=4y$.

26. Using elementary row transformation. Find the inverse of the matrix $A=\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of lines equations:

$$8x+4y+3z=19$$
$$2x+y+z=5$$
$$x+2y+2z=7$$

27. Find the co-ordinates of the point P where the line through $A(3,-4,-5), B(2,-3,1)$ crosses the plane, passing through the points $(2,2,1),(3,0,1),(4,-1,0)$. Also, find the ratio in which P divides the line segment AB.

OR

Find the equation of the plane which contains the line of intersection of the planes.

$$\vec{r}\cdot\left(\hat{i}-2\hat{j}+3\hat{k}\right)-4=0$$
$$\text{and } \vec{r}\cdot\left(-2\hat{i}+\hat{j}+\hat{k}\right)+5=0$$

and whose intercept on x-axis is equal to that of on y-axis.

28. Find the particular solution of the differential equation:

$$xe^{\frac{y}{x}}-y\sin\left(\frac{y}{x}\right)+x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)=0 \text{ for } x=1, y=0$$

29. In order to supplement daily diet, a person wishes to take X and Y tablets. The contents (in mg/tablet) of iron, calcium and vitamins in X and Y are given below:

Tablets	Iron	Calcium	Vitamins
X	6	3	2
Y	2	3	4

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins.

The price of each tablet of X and Y is Rs.2 and Rs.1, respectively. How many tablets of each type should the person take in order to satisfy the above requirement at ht minimum cost?

Make an L.P.P and solve graphically.