

**General Instructions :**

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.
- (v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.
- (vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

**SECTION A**

1. For what value of  $k$ , the matrix  $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$  is a skew symmetric matrix?
2. Evaluate :  $\int \frac{1}{\sin^2 x \cos^2 x} dx$
3. If a line makes angles  $\alpha, \beta, \gamma$  with the positive direction of co-ordinate axes, then write the value of  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ .

**OR**

Find the vector equation for the line which passes through the point  $(1, 2, 3)$  and is parallel to the line

$$\frac{x-1}{-2} = \frac{1-y}{3} = \frac{3-z}{-4}$$

4. Find the value of  $c$  in Rolle's theorem for the function:  $f(x) = x^3 - 3x$  in  $[-\sqrt{3}, 0]$

**SECTION B**

5. If  $y = \sin^{-1}(6x\sqrt{1-9x^2})$ ,  $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$ , then find  $\frac{dy}{dx}$
6. Find the equation(s) of the tangent(s) to the curve  $y = (x^3 - 1)(x - 2)$  points where the curve intersects the  $x$ -axis.
7. If the equation of a line is  $x = ay + b; z = cy + d$ , then find direction ratios of the line and a point on the line.
8. Write in the simplest form;  $\sin^{-1} \left[ \frac{x + \sqrt{1-x^3}}{\sqrt{2}} \right]$ ,  $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$
9. Form the differential equation of equation  $y = a \cos 2x + b \sin 2x$ , where  $a$  and  $b$  are constants.

**OR**

Solve the differential equation  $\frac{dy}{dx} = \frac{1 + \cos 2y}{1 + \cos 2x}$

10. For what values of  $\lambda$ , is the function

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$

Is continuous at  $x=0$ ? What about continuity at  $x=1$ ?

OR

If  $x = \frac{at}{1+t^2}$ ,  $y = \frac{at^2}{1+t^2}$ , find  $\frac{dy}{dx}$  at  $t=2$

11. Evaluate:  $\int_0^{\frac{\pi}{2}} \log \tan x \, dx$

12. If  $A = \begin{bmatrix} 0 & b & -2 \\ 3 & 1 & 3 \\ 2a & 3 & -1 \end{bmatrix}$  is symmetric matrix, find the values of  $a$  and  $b$

OR

Solve the matrix equation  $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3 \begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$

SECTION C

13. Let R be a relation defined on the set of natural numbers N as follow:

$$R = \{(x, y); x \in N, y \in N \text{ and } 2x + y = 24\}$$

Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

14. If  $a, b, c$  are positive and unequal, show that the following determinant is negative.

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

15. Evaluate :  $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 \, dx$

OR

Find :  $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} \, dx$

16. The two adjacent sides of parallelogram are  $2\hat{i} - 4\hat{j} - 5\hat{k}$  and  $2\hat{i} + 2\hat{j} + 3\hat{k}$ . Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.
17. If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , ( $x \neq y$ ), then prove that  $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

OR

If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$ , then find  $\frac{d^2x}{dt^2}$ ,  $\frac{d^2y}{dt^2}$  and  $\frac{d^2y}{dx^2}$

18. Find the shortest distance between the lines  $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$   
and  $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

19. Evaluate  $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$

OR

Evaluate  $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$

20. Prove that  $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$
21. Solve the differential equation :  $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^3$
22.  $A$  and  $B$  throw a pair of dice alternatively.  $A$  wins the game if he gets a total of 9 and  $B$  wins if he gets a total of 7. If  $A$  starts the game, find the probability of winning the game by  $B$ .
23. Find the vector  $\vec{p}$  which is perpendicular to both  $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$  and  $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{p} \cdot \vec{q} = 21$ , where  $\vec{q} = 3\hat{i} + \hat{j} - \hat{k}$ .

SECTION D

24. Using elementary column operation, find the inverse of the following matrix

$$\begin{vmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

OR

Using matrices, solve the following system of linear equations:

$$x - y + 2z = 7, 3x + 4y - 5z = -5, 2x - y + 3z = 12.$$

25. An open box, with a square base is to be made out of a given quantity of metal sheet of area  $c^2$ . Show that the maximum volume of the box is  $\frac{c^2}{6\sqrt{3}}$ .

OR

The sum of surface areas of a sphere and a cuboid with sides  $\frac{x}{3}$ ,  $x$  and  $2x$  is constant. Show that the sum of their volumes is minimum if  $x$  is equal to the three times the radius of sphere.

26. Using integration, find the area of the region bounded by the line  $x - y + 2 = 0$ , the curve  $x = \sqrt{y}$  and Y axis.

OR

Find the area of the region bounded by the region enclosed by curves.

$$(x - 6)^2 + y^2 = 36 \text{ and } x^2 + y^2 = 36$$

27. Find the equation of the plane through the line of intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  and twice of its y-intercept is equal to three times its z-intercept.
28. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let be the probability that he knows the answer and  $\frac{2}{3}$  be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability  $\frac{1}{3}$ , what is the probability that the student knows the answer given that he answered it correctly.
29. There are two types of fertilizers 'A' and 'B', 'A' consists of 12% nitrogen and 5% phosphoric acid 'B' consists of 4% nitrogen and 5% Phosphoric acid. After testing the solid conditions, farmer finds that he need at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs Rs.10 per kg and 'B' costs Rs.8 per kg, then graphically determine how much of each type of fertilizer should be used so that nutrient requirements are at a minimum cost.