MATHEMATICS **PAPER - 20**

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.

(v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.

(vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

SECTION A

1. Find x from the matrix equation

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

2. If a line makes angles 90°, 60° and θ with x, y and z-axis respectively, where θ is an acute angle, then find θ.

OR

Find the direction cosines of the line

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

3. Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^3 x \, dx$

4. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

SECTION B

5. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}} dx$

6. Find the value of p for which the function

$$f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & x \neq 0 \\ p, & x = 0 \end{cases}$$

Is continuous at $x = 0$.

OR

Find $\frac{dy}{dx}$ at $x = 1$, $y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = k$

7. If $x = \theta \sin\theta, y = \theta\cos\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

8. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y increasing at the rate of 4 cm/min, find the rate of change of its area when x = 5 cm and $y = 8\,cm$.

9. Given that $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}\times\vec{b} = \vec{0}$, what can you conclude about the vector $\vec{a}$ and $\vec{b}$?

10. Find $\frac{1}{2}\left(A = A^T\right)$ and $\frac{1}{2}\left(A - A^T\right)$. If

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

OR

If $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, find the value of θ satisfying the equation $A + A^T = I_2$, where $0 \le \theta \le \frac{\pi}{2}$.

11. If $P(E) = \frac{1}{2}$ and $P(F) = \frac{1}{5}$, find $P\left(\overline{E \cup F}\right)$, if E and F are independent events.

12. Two tailors, A and B, earn Rs.300 and Rs.400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce to least 60 shirts and 32 pairs of trousers at a minimum labour cost. Formulate this as an L.P.P.

OR

Solve the following Linear programming Problem graphically:
Maximize $Z = 3x + 4y$
Subject to $x + y \le 4$, $x \ge 0$ and $y \ge 0$

SECTION C

13. Using properties of determinants, prove that

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xy\,(x+y+z)^3$$

OR

Using properties of determinants, prove that following

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

14. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^3}=2xyz$

15. Find $\frac{dy}{dx}$, *if* $y=(\cos x)^x+(\sin x)^{1/x}$

OR

If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then prove that

$$(1-x^2)\frac{d^w y}{dx^2}-3x\frac{dy}{dx}-y=0$$

16. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lit. Find the mean and variance of the distribution.

17. Find the Cartesian as well as vector equations of the planes through the intersection of planes. $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$, which are at a unit distance from the origin.

18. Find : $\int\frac{\sin x}{\sin^3 x+\cos^3 x}dx$

19. Solve the following differential equation

$$\left[y-x\cos\left(\frac{y}{x}\right)\right]dy+\left[y\cos\left(\frac{y}{x}\right)-2x\sin\left(\frac{y}{x}\right)\right]dx=0$$

OR

Show that the differential equation $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+x\,dy=0$ is homogeneous.

Find the particular solution of this differential equation, given that $y=\frac{\pi}{4}$ when $x=1$

20. Vectors $\vec{a},\vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$

21. Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B is found to be red. Find the probability that two red balls were transferred from A to B.

22. Evaluate: $\int_{-2}^{2}\frac{x^2}{1+5^x}dx$

23. A company manufactures two types of sweaters, type A and B. It costs Rs.360 to make one unit to type A and Rs.120 to make a unit of type B. The company can make at most 300 sweaters and can spend Rs.72,000 a day. The number of sweaters of type A cannot exceed

the number of type B be more than 100. The company makes a profit of Rs.200 on each unit of type A. The company is charging a nominal profit of Rs.20 on a unit of type B. Using L.P.P., solve for maximum profit.

SECTION D

24. Find the area of the region $\{(x,y): x^2+y^2 \le 4, x+y \ge 2\}$.

OR

Find the area of the region bounded by the two parabolas $y^2=4ax$ and $x^2=4ay$, when $a>0$.

25. From the differential equation of the family of circles touching the x-axis at origin.

26. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$$\vec{r}=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$$
$$\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$$

OR

Find the foot of perpendicular from $P(1,2,-3)$ to the line $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$. Also, find the image of P in the given line.

27. Show that the binary operation * on $A=R-\{-1\}$ defined as $a*b=a+b+ab$ for all $a,b,c.$ A is commutative and associative on A. Also, find the identity element of * in A and prove that every element of A is invertible.

28. If $A=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{bmatrix}$, find A^{-1} . Hence solve the system of equations: $x+2y+z=4, -x+y+z=0$ and $x-3y+z=4$

29. Show that of all the rectangle of given area, the square has the smallest perimeter.

OR

A tank with rectangular base and rectangular sides, open at the top is to be constructed, so that its depth is 2 m and volume is 8 m^3. If building of tank cost Rs.70 per sq m for the base and Rs.45 per sq.m. for sides. What is the cost of least expensive tank?