MATHEMATICS **PAPER-19**

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. Let $A=\{1,2,3,4\}$ and R be the equivalence relation on $A\times A$ defined by $(a,b)R(c,d)$ iff $a+d=b+c$. Find the equivalence class $[(1,3)]$.

2. If $A=[a_{ij}]$ is a matrix of order 2×2, such that $|A|=-15$ and c_{ij} represents the cofactor of a_{ij}, then find $a_{21}c_{21}+a_{22}c_{22}$.

3. Give an example of vectors $\vec{a}$ *and* $\vec{b}$ such that $|\vec{a}|=|\vec{b}|$ but $\vec{a}\neq\vec{b}$.

4. Determine whether the binary operation * on the set N of natural numbers defined by $a*b=2^{ab}$ is associative or not.

SECTION - B

5. If $4\sin^{-1}x+\cos^{-1}x=\pi$, then find the value of x.

6. Find the inverse of the matrix $\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

7. Prove that if $\frac{1}{2}\le x\le 1$, then $\cos^{-1}x+\cos^{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right]=\frac{\pi}{3}$.

8. Find the approximate change in the value of $\frac{1}{x^2}$, when x changes from $x=2$ to $x=2.002$.

9. Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx$.

10. Verify that $ax^2 + by^2 = 1$ is a solution of the differential equation $x\left(yy_2 + y_1^2\right) = yy_1$.

11. Find the projection (vector) of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.

12. If A and B are two events such that $P(A) = 0.4, P(B) = 0.8$ and $P(B/A) = 0.6$, then find $P(A/B)$.

SECTION - C

13. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$, then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$.

14. Find a and b, if the function given by $f(x) = \begin{cases} ax^2 + b, & if\ x < 1 \\ 2x+1, & if\ x \geq 1 \end{cases}$ is differentiable at $x = 1$.

OR

Determine the values of a and b such that the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} \frac{x + \sin x}{\sin(a+1)x}, & if\ -\pi < x < 0 \\ 2, & if\ x = 0 \\ \frac{2\left(e^{\sin bx} - 1\right)}{bx}, & if\ x > 0 \end{cases}$$

15. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

16. Find the equation(s) of the tangent(s) to the curve $y = \left(x^3 - 1\right)(x-2)$ at the points. Where the curve intersects the X-axis.

OR

Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$ is strictly increasing or strictly decreasing.

17. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the formula $C(x) = x^3 - 45x^2 + 600x$. Where x is the number of trees and C(x) is the cost of plating x trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? Use calculus to answer these questions. Which value is being exhibited by the person?

18. Find $\int \frac{\sec x}{1 + \cos ecx} dx.$

19. Find the particular solution of the differential $ye^y dx = \left(y^3 + 2xe^y\right)dy, y(0) = 1$.

OR

Show that $(x - y)dy = (x + 2y)dx$ is a homogeneous differential equation. Also, find the general solution of the given differential equation.

20. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ and hence show that $\left[\vec{a}\vec{b}\vec{c}\right] = 0.$

21. Find the equation of the line which intersects the lines
$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4} \, and \, \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point $(1,1,1)$.

22. Bag I contains 1 white, 2 black and 3 red balls; bag II contains 2 white, 1 black and 1 red balls; bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement.
They happen to be one white and one red. What is the probability that they came from bag III?

23. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution.

SECTION - D

24. If the function $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ by $g(x) = x^3 + 5,$ then find fog and show that fog is invertiable. Also, find $(fog)^{-1}$ and hence find $(fog)^{-1}(9).$

OR

A binary operation * is defined on the set R of real numbers by a*b $= \begin{cases} a, & if\ b=0 \\ |a|+b, & if\ b \neq 0 \end{cases}$. If at least one of a and b is 0, then prove that $a*b=b*a$. Check whether * is commutative. Find the identity element for *., if it exists.

25. If $A=\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations.

$3x+4y+7z=14, 2x-y+3z=4, x+2y-3z=0$

OR

If $A=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, then find the inverse of a using elementary row transformations and hence solve the matrix equation $XA=\left[101\right]$.

26. Using integration, find the area in the first quadrant bounded by the curve $y=x|x|$, the circle $x^2+y^2=2$ and the Y-axis.

27. Evaluate $\int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$.

OR

Evaluate $\int_{-2}^{2} \left(3x^2-2x+4\right) dx$ as the limit of a sum.

28. Find the distance of point $-2\hat{i}+3\hat{j}-4\hat{k}$ from the line $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda\left(\hat{i}+3\hat{j}-9\hat{k}\right)$ measured parallel to the plane $x-y+2z-3=0$.

29. A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and `1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and `0.9 in raw matrials. Given the current availability of staff in the company, each day there is at most a total of 90 h available for assembly and 80 h for quality control. The first product described has a market value (sale price) of `9 per unit and the second product described has a market value (sale price) of `8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

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