MATHEMATICS **PAPER 18**

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General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short answer type questions carrying 2 mark each.

(v) Question 13-23 in Section C are long answer I type questions carrying 4 mark each.

(vi) Question 24-29 in Section D are long answer II type questions carrying 6 mark each

SECTION A

1. If A is an invertible matrix of order 3 × 3 and $|A| = 9$, n find $|A \cdot \text{adj } A|$.

2. Find the value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

3. Consider the binary operation * on Q defined as $a * b = a + 5b + ab\ \forall a, b \in Q$. Find $6 * \frac{1}{5}$.

4. For ay vector $\vec{a}$, show that

$$\hat{i} \times \left(\vec{a} \cdot \hat{k}\right)\hat{j} + \hat{j} \times \left(\vec{a} \cdot \hat{i}\right)\hat{k} + \hat{k} \times \left(\vec{a} \cdot \hat{j}\right)\hat{i} = \vec{a}$$

OR

If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, find a unit vector in the direction of $\vec{a} - \vec{b}$

SECTION B

5. Prove that : $\cot\left[\frac{\pi}{4} - 2\cot^{-1} 3\right] = 7$

6. A square matrix A satisfies $A^2 = I - A$, where I is the identity matrix.

If $A^n = 5A - 3I$, find the value of n.

7. If $y = \tan^{-1}\left[\frac{5ax}{a^2 - 6x^2}\right]$, then prove that $\frac{dy}{dx} = \frac{3a}{a^2 + 9x^2} + \frac{2a}{a^2 + 4x^2}$

OR

If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$, prove that $(2y - 1)\frac{dy}{dx} = 1$

8. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

9. Evaluate : $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$

10. From the differential equation of the family of circles touching the x-axis at the origin.

OR

Solve the differential equation : $e^{x+y}-1=\frac{dy}{dx}$

11. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

12. If $2P(A)=P(B)=\frac{5}{13}$ and $P(A/B)=\frac{2}{5}$, evaluate $P(A\cup B)$

OR

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that one is club and the other is a king of red colour.

SECTION C

13. Using properties of determinants, prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

14. Find the values of *a* and *b* so that the function

$$\begin{cases} x+a\sqrt{2}\sin x & 0\le x<\frac{\pi}{4} \\ 2x\cot x+b & \frac{\pi}{4}\le x\le\frac{\pi}{2} \\ a\cos 2x-b\sin x & \frac{\pi}{2}<x<\pi \end{cases}$$ is continuous for $0\le x\le\pi$

OR

Show that the function $f(x)=|x-1|+|x-2|$ is not differentiable at $x=2$

15. If $(a+bx)e^{y/x}=x$, prove that $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$

16. Show that the equation of normal at any point on the curve $x=3\cos\theta-\cos^3\theta, y=3\sin\theta-\sin^3\theta$ is $4(y\cos^3\theta-x\sin^3\theta)=3\sin 4\theta$

OR

Find the intervals in which the function $f(x)=\sin x+\cos x, 0\le x\le 2\pi$ strictly increasing or strictly decreasing.

17. Find the dimensions of the rectangle with perimeter 36 cm which is rolled to form a cylinder of volume as large as possible with one of its sides Also, find the maximum volume of the cylinder.

18. Find the particular solution of the differential equation:

$$x^2\frac{dy}{dx}-xy=1+\cos\left(\frac{y}{x}\right), x\neq 0 \text{ and } y(1)=\frac{\pi}{2}$$

OR

Form the differential equation of the family of circles having radii 3 units.

19. Evaluate : $\int \frac{1}{\sin x\,(5-4\cos x)}dx$

20. If the three mutually perpendicular vectors $\vec{a},\vec{b},\vec{c}$ with $|\vec{a}|=1, |\vec{b}|=3$ and $|\vec{c}|=5,$ form a right handed system, then prove that
$(\vec{a}-2\vec{b})\cdot\left[(\vec{b}-3\vec{c})\times(\vec{c}-4\vec{a})\right]=-215$

21. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ angles of $\frac{\pi}{3}$ each.

22. A bag contains $(2n+1)$ coins. It is known that n of these coins have heads on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss result in the head is $\frac{31}{42}$, determine the value of n.

23. On a multiple choice examination with four possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers.

SECTION D

24. Consider the mapping $f:[0,2]\to[0,2]$ defined by $f(x)=\sqrt{4-x^2}$. Show that f is invertible and hence find f^{-1}.

OR

Let N be the set of all natural numbers and R be a relation on $N\times N$ defined by $(a,b)\,R(c,d)\Rightarrow ad=bc$ for all $(a,b),(c,d)\in N\times N$. Show that R is an equivalence realation on $N\times N$.

25. Find the matrix A satisfying the matrix equation
$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

OR

If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the system of equations

$x+2y+z=4$
$-x+y+z=0$
$x-3y+z=2$

26. Find the area of the part of the circle $x^2 + y^2 = 36$ which is exterior to the parabola $y^2 = 9x$

27. Evaluate : $\int_0^{1/2} \frac{1}{\left(1+x^2\right)\sqrt{1-x^2}} dx$

OR

Evaluate : $\int_0^4 \left(x+e^{2x}\right) dx$ as the limit of the sum

28. Find the distance of the point $P(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line $\frac{x+1}{2}=\frac{y+3}{3}=\frac{z+1}{-6}$

29. If a young man rides his motorcycle at 25 km/hour he had to spend Rs.2 per km on petrol. If he rides at a faster speed of 40 km/hour the petrol cost increases to Rs. 5 per km. He has Rs.100 to spend on petrol and wishes ot find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically and find the maximum distance.