MATHEMATICS **PAPER-17**

General Instruction:

1. **All questions are compulsory.**
2. **This question paper contains 29 questions.**
3. **Question 1-4 in Section A are very short answer type questions carrying 1 mar each.**
4. **Questions 5-12 in Section B are short answer type questions carrying 2 marks each.**
5. **Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.**
6. **Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.**
7. **There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.**

Time- 3 Hours **[Max. Marks-100]**

SECTION -A

1. Evaluate $\int e^{2x^2+\ln x}dx.$

2. For what value of k, the matrix $\begin{bmatrix} 2-k & 4 \\ -5 & 1 \end{bmatrix}$ is not invertible?

3. If $|x|<1$ and $f(x)=1+x+x^2+.....\infty$, then find the value of $f'(x)$.

4. Find the area of the parallelogram having adjacent sides $\vec{a}\ and\ \vec{b}$ given by $2\hat{i}+\hat{j}+\hat{k}\ and\ 3\hat{i}+\hat{j}+4\hat{k}$, respectively.

SECTION - B

5. Find the equation of the plane through the point $P(1,4,-2)$ and it is parallel to the plane $-2x+y-3z=0$.

6. Is the function $f(x)=\begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1}, & if\ x\neq 0 \\ 0, & if\ x=0 \end{cases}$ continuous at x=0?

7. Find the interval in which the function f given by $f(x)=2x^2-3x$ is strictly increasing.

8. Using differentials, find the approximate value of $\sqrt{49.5}$

9. Show that a matrix is both symmetric as –well as skew –symmetric is a null matrix.

10. Differentiate $\tan^{-1}\left(\frac{x}{1+6x^2}\right)$ with respect to x.

11. Evaluate $\int_1^e \frac{\cos(\log_e x)}{x}\,dx$.

12. Two events A and B are such that $P(A)=\frac{1}{2},\ \ P(B)=\frac{1}{3}\ and\ P(A\cup B)=\frac{2}{3}$. Are the events A and B mutually independent?

SECTION - C

13. Separate $\left[0,\frac{\pi}{2}\right]$ into subintervals in which $f(x)=\sin 3x$ is

(a) increasing (b) decreasing

14. Square piece of tin of side 18 cm is to be made into a box without a top by cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off, so that the volume of the box be maximum? Also, find the maximum volume of the box.

15. Find the equation of the curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

16. Prove that $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1-x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}=\left(\frac{x+y}{1-xy}\right), where |x|<1, y>0$ and $xy<1$.

17. Evaluate $\int \frac{2+\sin x}{1+\cos x}e^{x/2}dx.$

18. Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx.$

OR

Prove that $\int_0^{\pi} \frac{x}{(1+\sin x)}dx=\pi$

19. A school wants to award its students for the values of Honesty, Regularity and Hardwork with a total cash award of ₹6000. Three times the award money for Hardwork added to that given for Honesty amounts to ₹11000. The award money given for Honesty and Hardwork together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, by using matrix method. Apart from these values, namely, Honesty, Regularity and Hardwork, suggest one more value which the school must include for awards.

20. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

21. Find the coordinates of the point, where the line through $(3,-4,-5)\,and\,(2,-3,1)$ crosses the plane $2x+y+z=7$.

OR

Find the foot of perpendicular from the point (2, 3, 4) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find the length of the perpendicular segment.

22. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 yr. One student is selected in such a manner that each has the same chance of being of chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

OR

3 bad eggs are mixed with 7 good eggs. 3 eggs are taken at random from the 10 eggs. Find the probability distribution of number of bad eggs drawn. Also, find the mean and variance of the probability distribution.

23. In an electric bulb factory, machines A, B and C manufacture 60%, 30% and 10% electric bulbs, respectively. 1% , 2% and 3% of the electric bulbs produced respectively by A, B and C are found to be defective. An electric bulb is picked up at random from the total production and found to be defective. Find the probability that defective electric bulb was produced by machine A.

SECTION - D

24. A dealer in rural area wishes to purchase a number of sewing machines. He has only `5760 to invest and has space for atmost 20 items for storage. An electronic sewing machine cost him `360 and a manually operated sewing machine `240. He can sell, and electronic sewing machine at a profit of `22 and a manually operated sewing machine at a profit of `18.

Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise his profit? Make it as an LPP and solve it graphically. Keeping the rular background in mind to justify the values to be promoted for the selection of the manually operated machine.

25. Show that the relation R in the S at $A=\{x: x\in W, 0\le x\le 12\}$ given by $R=\{(a,b): |\text{a}-\text{b}| \text{is multiple of } 4\}$ is an equivalence relation. Also, find the set of all elements related to 2.

OR

Show that the function $f: R\to\{\text{x}\in\text{R}: -1<\text{x}<1\}$ defined by $f(x)=\frac{x}{1+|\text{x}|}, x\in R$ is one-one and onto function.

26. Using matrices, solve the following system of equations. $x+y=5, y+z=3 \, and \, z+x=4$

27. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$

OR

Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.

28. Find the area of the region $\{(x,y): 0\le y\le x^2+1, 0\le y\le x+1, 0\le x\le 2\}$.

29. The rate of increases of bacteria in a culture is proportional to the number of bacteria present. If the original number of bacteria doubles in two hours, in how many hours will it be five times?

OR

At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4,-3)$. Find the equation of the curve given that it passes through $(-2,1)$.

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