MATHEMATICS PAPER-16

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. Evaluate the determinant $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$.

2. If a unit vector $\hat{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with $\hat{j}$ and an acute angle γ with $\hat{k}$, find the value of γ.

3. Write the value of a for which $f(x) = \begin{cases} 5x-4, & 0 < x \le 1 \\ 4x^2 + 3ax, & 1 < x < 2 \end{cases}$ is continuous at x=1?

4. Evaluate $\int_0^2 e^{x-[x]} dx$.

SECTION - B

5. If $x^m y^n = (x+y)^{m+n}$, then prove that $\frac{d^2y}{dx^2} = 0$.

6. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three non-coplanar vectors, prove that $\left[\vec{a}+\vec{b}+\vec{c} \;\; \vec{a}+\vec{b} \;\; \vec{a}+\vec{c}\right] = -\left[\vec{a}\vec{b}\vec{c}\right]$.

7. If $A = \begin{bmatrix} 3 & 5 \\ 7 & -9 \end{bmatrix}$ *and* $B = \begin{bmatrix} 6 & -4 \\ 2 & 3 \end{bmatrix}$, find $(4A - 3B)$.

8. Show that the function $y = Ax + \frac{B}{x}$ is a solution of the differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

9. An edge of a varibal cube in increasing at the rate of 10 cm/s. How fast the volume of the cube is increasing when the edge is 5 cm long?

10. Find the approximate value of $\sqrt{25.2}$.

11. Evaluate $\int_{-1}^{2}\left(|x+1|+|x|+|x-1|\right)dx$.

12. Find the binomial distribution for which the mean is 4 and variance 3.

SECTION - C

13. If $f(x)\, and\, g(x)$ are two functions derivable in $[a,b]$ such that $f(a)=4,$ $f(b)=10, g(a)=1$ and $g(b)=3,$ show that for $a<c<b, f'(c)=3g'(c)$.

14. For which value of k, is the function $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2}, & if\ x\neq 0\\ k, & if\ k=0\end{cases}$ continuous at $x=0$?

15. Find the adjoint of the matrix $a=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and hence show that $A(adjA)=|A|I_3$.

16. Prove that $\tan^{-1}\left(\dfrac{a\cos x-b\sin x}{b\cos x-a\sin x}\right)=\tan^{-1}\left(\dfrac{a}{b}\right)-x.$

OR

Prove that $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\dfrac{x}{2}, x\in\left(0,\dfrac{\pi}{4}\right).$

17. Find the particular solution of the following differential equation

$xy\dfrac{dy}{x}=(x+2)(y+2); y=-1,$ when $x=1.$

OR

Find the particular solution of the following differential equation.

$x\left(x^2-1\right)\dfrac{dy}{dx}=1; y=0, when\, x=2$.

18. Evaluate $\int\dfrac{dx}{\sin^4 x+\cos^4 x}$.

19. Evaluate $\int_0^{\pi}\dfrac{dx}{3+2\sin x+\cos x}$.

20. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors, such that $|\vec{a}| = 5, |\vec{b}| = 12, |\vec{c}| = 13$ *and* $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

21. Find the vector equation of the plane passing through the point $(2, 0, -1)$ and perpendicular to the line joining the points $(1, 2, 3)$ and $(3, -1, 6)$.

OR

Find the equation of the line passing through the point $(2, 1, 3)$ and perpendicular to the lines $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{3}$ *and* $\frac{x-4}{-3} = \frac{y+1}{2} = \frac{z-1}{5.}$.

22. In a box containing 100 bulbs, 10 are defective. What is the probability that out of sample of 5 bulbs, none is defective? Write two advantages of using CFL (compact fluroscent lamp) bulbs over incandescent bulbs.

23. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Out of their output 3%, 4% and 1%, respectively are defective bolds. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

SECTION - D

24. A binary operation '*' is defined on the set $X = R - \{-1\}$ by $x * y = x + y + xy, \forall x, y \in X$. Check whether '$*$' is commutative and associative. Find its identity element and also find the inverse of each element of X.

OR

If N denotes the set of all natural number and R be the relation on $N \times N$ defined by $(a, b) R (c, d), if\ ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

25. A manufacture considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at ₹100 and ₹120 per unit respectively, how should he use his resources to maximise the total revenue?
From the above as an LPP and solve graphically. What quality of manufacture shown here.

26. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

27. Using integration, find the area of the triangular region whose vertices are $P(1,0), Q(2,2)\,and\,R(3,1)$.

OR

Using integration find the area of the following region.

$\left\{(x,y) : |x-1| \le y \le \sqrt{5-x^2}\right\}$.

28. Find the vector equation of the plane passing through the three points with position vectors $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=\left(3\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$.

OR

Find the vector equation of the plane through the points $(2,1,-1)\,and\,(-1,3,4)$ and perpendicular to the plane $x-2y+4z=10$.

29. If a, b and c are all distinct and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix}=0$.

Show that $abc(ab+bc+ca)=a+b+c.$

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