MATHEMATICS **PAPER-15**

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. Find the additive inverse of the matrix $a=\begin{bmatrix}2 & -5 & 0\\ 4 & 3 & -1\end{bmatrix}$.

2. What are the direction cosines of a line which makes equal angles with the coordinate axes?

3. A ladder 5 m long, standing on a horizontal floor, leans against a vertical wall. If the two of the ladder slides downwards at the rate of 10 cm/s, then find the rate at which the angle between the floor and ladder is decreasing when the lower end of ladder is 2m from the wall.

4. Evaluate: $\int e^{ax}\{\mathrm{af}(x)+\mathrm{f}'(x)\}dx$.

SECTION - B

5. Evaluate $\int \dfrac{x^3}{x^4+3x^2+2}dx.$

6. Find the values of λ and μ for which $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$.

7. If $f(x)=\begin{cases}\dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}, & x\neq 0\\ a, & x=0\end{cases}$ is continuous at $x=0$, find the value of a.

8. Prove that the determinant $\begin{vmatrix}x & \sin\theta & \cos\theta\\ -\sin\theta & -x & 1\\ \cos\theta & 1 & x\end{vmatrix}$ is independent of θ.

9. If $f(x)=x^3+ax^2+bx+c$ has maximum value at $x=-1$ and minimum at x=3. Determine the values of $a, b\, and\ c$.

10. If $y=\frac{5^x}{x^5}$, then find $\frac{dy}{dx}$.

11. Find the approximate value of $(1.999)^5$.

12. If $A=\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} and\ B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then find the value of x for which $A^2=B$.

SECTION - C

13. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$.

14. Show that $x=2$ is a root of the equation formed by the following determinant.

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}=0$ Hence, solve the equation.

15. If $y=a^{x^{a^{x^{\cdots\infty}}}}$, prove that $\frac{dy}{dx}=\frac{y^2\log y}{x[1-y\log x\log y]}$.

16. Evaluate $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$.

OR

Evaluate $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$.

17. Evaluate the integral $\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$.

18. Solve the differential equation $(xdy-ydx)y\sin\left(\frac{y}{x}\right)-(ydx+xdx)x\cos\left(\frac{y}{x}\right)=0$.

19. Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1, x\neq 0$.

20. Express the vector $\vec{a} = \left(5\hat{i} - 2\hat{j} + 5\hat{k}\right)$ as sum of two vectors such that one is parallel to the vector $\vec{b} = \left(3\hat{i} + \hat{k}\right)$ and the other is perpendicular to $\vec{b}$.

21. Find the angle between the line $\dfrac{x+1}{2} = \dfrac{y}{3} = \dfrac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

OR

Find the equation of the plane which contains the line of intersection of planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$, $\vec{r}.\left(2\hat{i} + \hat{j} + \hat{k}\right) - 15 = 0$ and in perpendiculars to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

22. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

OR

Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'that die shows a number greater than 3' given that 'there is atleast one head'.

23. To promote the making of toilets for women, and organization tried to generate awareness through
(i) house calls (ii) letters (iii) announcements
The cost for each mode per attempt is given below
(i) ₹50 (ii) ₹20 (iii) ₹40
The number of attempts made in three villages X, Y and Z are given below

Village	House calls	Letters	Announcements
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the cost incurred by the organization for X and Z villages separately using matrices. Write one value generated by the organisation in the society.

SECTION - D

24. If $f : R - \{2\} \rightarrow \text{R} - \{3\}$ is defined by $f\left(x\right) = \dfrac{3x+1}{x-2}$, where R is the set of real numbers, show that f is invertiable and hence find the value of f^{-1}.

OR

Let $f : N \rightarrow R$ be a function defined as $f\left(x\right) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow$ range(f) is invertible. Find the inverse of *f*.

25. A point on the hypotenuse of a right angled triangle is at distance of a units and b units from the sides. Show that the minimum length of hypotenuse is $\left(a^{2/3}+b^{2/3}\right)^{3/2}$.

OR

It the straight line $x\cos\alpha+\text{y}\sin\alpha=\text{P}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ prove that $p^2=a^2\cos^2\alpha+b^2\sin^2\alpha$.

26. A farmer has a supply of chemical fertilizers of type a which contains 10% nitrogen and 6% phosphoric acid and type B contains 5% of nitrogen and 10% of phosphoric acid. After soil testing, it is found that atleast 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizers of types A and B cost `5 and `8 per kg, respectively. By using LPP, find how many kilograms of each type of fertilizers should be bought to meet the requirement and cost be minimum? Solve the problem graphically.
We generally deal with two types of farming
(i) Farming using fertilizers
(ii) Organic farming
Which farming you would prefer and why?

27. Find the image of the point $2\hat{i}+3\hat{j}-4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=3.$

OR

Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5,$ which is perpendicular to the plane $x-y+z=0$. Also, find the distance of the plane obtained above from the point (1, 1, 1).

28. Find the area lying above the X-axis and included between the circle $x^2+y^2=8x$ and parabola $y^2=4x.$

29. Let d_1, d_2 and d_3 be three mutually exclusive diseases and $S=\left(s_1,s_2,s_3,......,s_6\right)$ be the set of observable symptoms of these disease. For example, s_1 is the shortness of breath, s_2 is loss of weight, s_3 is fatigue, etc. Suppose a random sample of 10000 patients contains 3200 patients with disease d_1, 3500 with disease d_2 and 3000 with disease d_3 show the symptoms S. Knowing that the patient has symptom S, the doctor wishes to determine the patient's illness. On the basis of this information, what should the doctor conclude?

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