MATHEMATICS **PAPER-14**

General Instruction:

1. ***All questions are compulsory.***
2. ***This question paper contains 29 questions.***
3. ***Question 1-4 in Section A are very short answer type questions carrying 1 mar each.***
4. ***Questions 5-12 in Section B are short answer type questions carrying 2 marks each.***
5. ***Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.***
6. ***Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.***
7. ***There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***

Time- 3 Hours ***[Max. Marks-100]***

SECTION -A

1. If A is an invertible matrix of order 3×3 such that |A|=5, find the value of $\left|A^{-1}\right|$.

2. If A, B and C are the vertices of a ΔABC, what is the value of $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$?

3. The sides of an equilateral triangle are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when the side is 10 cm.

4. Evaluate $\int \frac{1}{x(\log x)^n}dx.$

SECTION - B

5. Find the integrating factor of differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$.

6. Find the projection of vector $2\hat{i}+\hat{j}$ on the vector $\hat{i}+2\hat{j}$.

7. Let A and B be square matrices of the order 3×3. Is $(AB)^2=A^2B^2$? Given reason.

8. If $x^{16}y^9=\left(x^2+y\right)^{17}$, then prove that $x\frac{dy}{dx}=2y$.

9. Find the value of m and n, where m and n are order and degree of differential equation $\dfrac{4\left(\dfrac{d^2y}{dx^2}\right)^3}{\dfrac{d^3y}{dx^3}}+\dfrac{d^3y}{dx^3}=x^2-1$.

10. Find the approximate value of $\dfrac{1}{\sqrt{25.1}}$.

11. If the slope of the curve $2y^2=ax^2+b$ at $(1,-1)\, is -1,$ find the values of a and b.

12. There are 5 white and 8 red balls in bag A, 7 white and 6 red balls in bag B, 6 white and 5 red balls in bag C. One ball is taken out at random from each bag. Find the probability that all three balls are of the same colour.

SECTION - C

13. A total amount of `7000 is deposited in three different saving bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$, respectively. The total annual interest from these three accounts is `550. Equal amounts have been deposited in the 5% and 8% saving accounts. Find the amount deposited in each of the three accounts, with the help of matrix multiplication. Keeping notion's growth in mind, justify the value of saving in individual life.

14. Find the intervals in which $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is

(a) strictly increasing (b) strictly decreasing

15. Differentiate $\tan^{-1}\left(\dfrac{a\cos x-b\sin x}{b\cos x+a\sin x}\right),\ \dfrac{-\pi}{2}<x<\dfrac{\pi}{2}\ and\ \dfrac{a}{b}\tan x>-1$ w.r.t.x.

16. Prove that $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$.

17. Evaluate $\int x^{2s}(1+\log x)\,dx.$

18. Evaluate $\int_0^{\pi/2}\log(\sin x)\,dx$

OR

Evaluate $\int\dfrac{\sin^6 x+\cos^5 x}{\sin^2 x\cos^2 x}dx$

19. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ *and* $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ *and* $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

20. Find the foot of perpendicular drawn from the point A(1, 8, 4) to the line joining the points $B(0,-1,3)$ *and* $C(2,-3,1)$.

21. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X=4)=P(X=2)$. Find the probability of success.

OR

A bag A contains 4 block and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

22. Two groups are competing for the position on the board of directors of a corporation. The probability that the first and second groups will win are 0.6 and 0.4, respectively. Further, if the first group wins the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product introduce was by the second group.

23. Show that the differential equation $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0$ is homogeneous. Find the particular solution of this differential equation, given that $y=\frac{\pi}{4}, when\ x=1$.

OR

Find the solution of differential equation $x^2dy+y(x+y)dx=0, if\ x=1 and\ y=1$.

SECTION - D

24. Consider the function $f:R^+\to[4,\infty)$ defined by $f(x)=x^2+4$, where R^+ is the set of all non negative real numbers. Show that f is invertible. Also, find the inverse of f f.

OR

Show that the relation S in the set $A\equiv\{x\in Z:0\le x\le 12\}$ given by S={ $(a,b):a,b\in Z,|a-b|$ is divisible by 4} is an equivalence relation. Find the set off all elements related to 4.

25. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

26. Find the area of region bounded by lines $y=\frac{5}{2}x-5, x+y-9=0\ and\ y=\frac{3}{4}x-\frac{3}{2}$.

27. Find the equation of plane determined by points $A(3,-1,2), B(5,2,4)$, $C(-1,-1,6)$ and hence find the distance between plane and point P(6, 5, 9).

OR

Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ are intersecting. Hence, find their point of intersecting.

28. A cooperative society of farmers has 50 hec of land to grow two crops A and B. The profit from crops A and B per hectare are estimated as `10500 and `9000, respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20L/hec and 10 L/hec, respectively. Further not more than 800 L, herbicide should be used in order to protect fish and wildlife using a pond which collect drainage from the land keeping in mind that the protection of fish and other wildlife is more important than earning profit. How much land should be allocated to each crop, so as to maximum the total profit? Formulate the above as an LPP and solve it graphically.

So you agree with the message that the protection of wild life is atmost necessary to preserve the balance in environment.

29. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the following equation.

$A^3 - 4A^2 + 11I - 3A = O$

OR

If $A + B + C = \pi$, show that

$$\begin{vmatrix} \sin^2 A & \sin A \cos A & \cos^2 A \\ \sin^2 B & \sin B \cos B & \cos^2 B \\ \sin^2 C & \sin C \cos C & \cos^2 C \end{vmatrix} = -\sin(A-B)\sin(B-C)\sin(C-A).$$

*****