MATHEMATICS **PAPER-13**

General Instruction:

1. **All questions are compulsory.**
2. **This question paper contains 29 questions.**
3. **Question 1-4 in Section A are very short answer type questions carrying 1 mar each.**
4. **Questions 5-12 in Section B are short answer type questions carrying 2 marks each.**
5. **Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.**
6. **Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.**
7. **There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.**

Time- 3 Hours **[Max. Marks-100]**

SECTION -A

1. If the vertices A, B, C of a ΔABC have position vectors $(1,2,3),(-1,0,0)$ and $(0,1,2)$ respectively, what is the magnitude of $\angle ABC$?

2. If $A=\begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}$, *find* A^{16}.

3. Evaluate $\int_0^1 \log\left(\frac{1-x}{x}\right)dx.$

4. If $x=a\sin mt-b\cos mt$ *and* $\frac{d^2x}{dt^2}=\mu x,$ then find the value of μ.

SECTION - B

5. If $A(x_1,y_1),B(x_2,y_2)$ *and* $C(x_3,y_3)$ are vertices of an equilateral triangle whose each side is equal to a, then prove that $\begin{vmatrix}x_1 & y_1 & 2\\ x_2 & y_2 & 2\\ x_3 & y_3 & 2\end{vmatrix}=3a^4$.

6. If $y=\tan^{-1}x,$ *find* $\frac{d^2y}{dx^2}$ in terms of y alone.

7. Find λ, where projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.

8. At what point on the curve $y=x^2$ does the tangent make an angle of 45° with the X-axis?

9. Examine the continuity of the function $f(x)=\begin{cases}\dfrac{|x-4|}{2(x-4)}, & if\ x\neq 4\\ 0, & if\ x=4\end{cases}$, which increases twice as fast as it sine.

10. Find an angle $\theta, 0<\theta<\dfrac{\pi}{2}$, which increases twice as fast as it sine.

11. Evaluate $\int \sin^4 x\cos^4 x dx$.

12. In a hurdles race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\dfrac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

SECTION - C

13. Two farmers Ramkishan and Gurcharan Singh cultivates only three crops rice, maize and wheat. The sales (in `) of these crops in the month of July and August are given by the following matrices.

July sales

	Rice	Maize	Wheat	
$A=$	1500	1500	50000	*Ramksirhan*
	45000	20000	25000	*Gurcharan Singh*

August sales

	Rice	Maize	Wheat	
$B=$	15000	10000	50000	*Ramkishan*
	45000	20000	25000	*Gurcharan*sin *gh*

(i) Find the combined sales in July and August for each farmer in each crop

(ii) If both farmers receive 1.5% profit on gross sales. Compute the profit for each farmer for crop sold in July

(iii) Give suggestions to the farmers keeping 'Save Environment' in mind.

14. Solve $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$.

15. Form the differential equation having $y=(\sin^{-1}x)^2+A\cos^{-1}x+B$, where A and B are arbitrary constants, as its general solution.

16. If $y=x\log\left(\dfrac{x}{a+bx}\right)$, then prove that $x^3\dfrac{d^2y}{dx^2}=\left(x\dfrac{dy}{dx}-y\right)^2$.

17. Evaluate $\int e^x\left(\dfrac{\sin 4x-4}{1-\cos 4x}\right)dx$.

18. Evaluate $\int_3^4 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}dx.$

OR

Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx.$

19. A company has two plants for manufacture scooters. Plant I manufacturing 70% of the scooters and plant II manufactures 30% of the scooters. At plant I, 30% of the scooters are maintaining pollution norms and at plant II, 90% of the scooters are maintaining pollution norms. A scooter is chosen at random and is found to be fit on pollution norms. Find the probability that it has come from plant II.

OR

Event A and B are such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and $P(not\ A\ or\ not\ B)=\frac{1}{4}$, State whether A and B are independent.

20. If $\vec{a},\vec{b}\ and\ \vec{c}$ are three vectors, such that $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{c}|=5$ and each one of these is perpendicular to the sum of other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

21. Find the coordinates of point on line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$, which are at a distance of 3 units from the point $(1,-2,3)$.

OR

Show that $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})\ and\ \vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ are coplanar. Also, find the equation of the plane containing them.

22. Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=2x+x^2\cot x\ (x\neq 0)$ given that $y=0$ where $x=\frac{\pi}{2}$.

23. A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced. 6 balls are drawn in this way. Find the probability that

(i) All balls were purple

(ii) Not more than w were pink

(iii) An equal number of purple and pink balls were drawn

(iv) Atleast one ball was pink

SECTION - D

24. Consider $f:\mathrm{R}^{+}\to[-9,\infty)$ given by $f(x)=5x^{2}+6x-9$. Prove that f is invertible with $f^{-1}(y)=\left(\frac{\sqrt{54+5y}-3}{5}\right)$ (where, R^{+} is the set of all positive real numbers.)

OR

Let '*' be a binary operation on the set {0, 1, 2, 3, 4, 5} defined as $a*b=\begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & \text{if } a+b\geq 6 \end{cases}$

Show that zero is the identity for this operation and each element a of the set is invertible with $b-a$, being the inverse of a.

25. If a young man rides has motor-cycle at 25 km per hour, he had to spend `2 per km on petrol with very little pollution in the air. If he rides if at a faster speed of 40 km per hour, the petrol cost increases to `5 per km and rate of pollution also increases. He has `100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

26. Find the equation of the plane passing through the line of intersection of planes $2x+y-z=3, 5x-3y+4z+9=0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

OR

Find the distance of the point $(-2,3,-4)$ from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

27. Using integration, find the area of the following region. $\left\{(x,y):|\,x-1\,|\leq y\leq\sqrt{5-x^{2}}\right\}$

28. Find the volume of the largest cylinder that can be inscribed in sphere of radius r.

OR

A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of its semicircular ends is $\pi:(\pi+2)$.

29. If $x+y+z=0$, prove that $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix}=xyz\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.