MATHEMATICS **PAPER-12**

<u>**General Instruction:**</u>

1. **All questions are compulsory.**
2. **This question paper contains 29 questions.**
3. **Question 1-4 in Section A are very short answer type questions carrying 1 mar each.**
4. **Questions 5-12 in Section B are short answer type questions carrying 2 marks each.**
5. **Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.**
6. **Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.**
7. **There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.**

Time- 3 Hours **[Max. Marks-100]**

SECTION -A

1. Find matrix X, if $X+\begin{bmatrix}4 & 6\\ -3 & 7\end{bmatrix}=\begin{bmatrix}3 & -6\\ 5 & -8\end{bmatrix}$.

2. Evaluate $\int_0^{\pi/2} x\sin x dx.$

3. Find the value of $\frac{dy}{dx}, if\ y=\cos\left(\sec^{-1}x+\cos ec^{-1}x\right), |x|\geq 1$.

4. Find the volume of a parallelepiped whose sides are given by $-3\hat{i}+7\hat{j}+5\hat{k}, -5\hat{i}+7\hat{j}-3\hat{k}$ and $7\hat{i}-5\hat{j}-3\hat{k}$.

SECTION - B

5. Examine the continuity of the function $f\left(x\right)=\begin{cases}\frac{|\sin x|}{x}, & x\neq 0\\ 1, & x=0\ at\ x=0\end{cases}$.

6. Evaluate $\begin{vmatrix}0 & xy^2 & xz^2\\ x^2y & 0 & yz^2\\ x^2z & zy^2 & 0\end{vmatrix}$.

7. Solve the following differential equation $\frac{dy}{dx}=1-x+y-xy$.

8. Show that the function given by $f\left(x\right)=\sin x$ is neither strictly decreasing nor strictly increasing in the interval $\left(0,\pi\right)$.

9. Find $\frac{dy}{dx}, if\ y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0<x<1$.

10. A speak truth in 30% of the case and B is 60% of the case. In what percentage of cases are likely to contradict each other in stating the same fact?

11. If $\vec{a}\ and\ \vec{b}$ are two vectors, then show that $\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}.\vec{b}\right)=a^2b^2$.

12. Find the value of $\int_{-\pi/4}^{\pi/4}\left(x^3+x^4+\tan^3 x\right)dx$.

SECTION - C

13. Use matrix product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations $x-y+2z=1, 2y-3z=1$ and $3x-2y+4z=2$.

OR

The sum of three numbers is 6. Twice the third number when added to the first number given 7. On adding the sum of the second and third numbers to thrice the first number, we get 12. Find the numbers, using matrix method.

14. Find the value of $\left\{2\cot^{-1}\left(-\frac{5}{12}\right)\right\}$.

15. Evaluate $\int\frac{\sqrt{x^2+\left[\log|x^2+1|-2\log|x|\right]}}{x^4}dx$.

16. An expensive square piece of golden color board of side 24 cm is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box. What should be the side of the square piece to be cut from each corner of the board to hold maximum volume and minimise the wastage? What is the importance of minimizing the wastage in utlising the resources?

17. Evaluate $\int\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}dx$.

18. Verify Rolle's theorem for the function $f(x)=\log\left\{\frac{x^2+ab}{x(a+b)}\right\} on [a,b]$, where $0<a<b$.

OR

Show that the function $f(x)=|x+1|+|x-1|$. $\forall x \in R,$ is not differentiable at the points $x=-1 \, and \, x=11$.

19. Find the equation of the plane passing through the points $(-1,2,0),(2,2,-1)$ and parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

20. Solve the differential equation $ye^{y}dx=\left(y^{3}+2xe^{y}\right)dy, y(0)=1$.

OR

Find the differential equation of the family of curves $y=e^{2x}(a\cos 2x+b\sin 2x)$, where $a \, and \, b$ are arbitrary constants.

21. A football match may be either won, drawn or lost by the host country's team. So, there are three ways of forecasting the result of any one match i.e one correct and two incorrect. Find the probability of forecasting atlest three correct results for four matches.

22. Simplify $\left[\vec{a}-\vec{b}\;\vec{b}-\vec{c}\;\vec{c}-\vec{a}\right]$.

23. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball does not depend on K.

SECTION - D

24. Show that the function $f: R \to R$ defined by $f(x)=\frac{3x-1}{2}, x \in R$ is one-one and onto functions. Also, find the inverse of the function f.

OR

Examine which of the following is a binary operation and check whether the operation is commutative and associative?

(i) On $Z^{*}, define \, a*b=2^{ab}$

(ii) On Q, define $a*b=\frac{ab}{2}$.

25. Two farmers Hariom and Siyaram cultivates only three varieties of rice namely Basmati, Permal and Naura. The sale (in `) of these varies of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September Sales (in `)

Basmati Permal Naura

$$A=\begin{bmatrix}10000 & 20000 & 30000\\ 50000 & 30000 & 10000\end{bmatrix}\begin{matrix}Hariom\\ Siyaram\end{matrix}$$

October sales (in `)

Basmati Permal Naura

$$B=\begin{bmatrix}5000 & 10000 & 6000\\ 20000 & 10000 & 10000\end{bmatrix}\begin{matrix}Hariom\\ Siyaram\end{matrix}$$

(i) Find the combined sales in September and October for each farmer in each variety

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October

Which variety of rice preferred the most and why?

26. Draw a rough sketch of $y^2=x+1$ and $y^2=-x+1$ and determine the area enclosed by the two curves.

OR

Draw a rough sketch of y=sin2x and determine the area enclosed by the curve, X-axis and lines $x=\frac{\pi}{4}\,and\,\frac{3\pi}{4}$.

27. Find the angle of intersection of the curves $y^2=4ax\,and\,x^2=4by$.

28. From the point P(1, 2, 4), a perpendicular is drawn on the plane $2x+y-2z+3=0$. Find the equation, the length and the coordinates of the foot of the perpendicular.

OR

Show that the lines

$\vec{r}=\left(-3\hat{i}+\hat{j}+5\hat{k}\right)+\lambda\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(-\hat{i}+2\hat{j}+5\hat{k}\right)+\mu\left(-\hat{i}+2\hat{j}+5\hat{k}\right)$ are coplanar.

Also, find the equation of the plane containing these lines.

29. There are two factories located at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are 8 and 6 units respectively.