MATHEMATICS PAPER-11

General Instruction:

1. **All questions are compulsory.**
2. **This question paper contains 29 questions.**
3. **Question 1-4 in Section A are very short answer type questions carrying 1 mar each.**
4. **Questions 5-12 in Section B are short answer type questions carrying 2 marks each.**
5. **Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.**
6. **Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.**
7. **There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.**

Time- 3 Hours **[Max. Marks-100]**

SECTION -A

1. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ....to\ \infty$, then prove that $\frac{d^2y}{dx^2} - y = 0$.

2. If A and B are matrices of order 3 and $|A| = 5, |B| = 3$, find $|3AB|$.

3. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

4. Evaluate $\int_0^1 3^{x-[x]} dx.$

SECTION - B

5. If $\vec{a}$ *and* $\vec{b}$ are the position vectors of a and B respectively, find the position vector of a point C on BA produced such that BC=1.5 BA.

6. Show that the function $f(x)$ given by $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0.$

7. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

8. Evaluate $\int \frac{1}{\sqrt{1+\sin 2x}} dx$.

9. Differentiate log sinx with respect to $\sqrt{\cos x}$.

10. If the radius of a circle is increased from 5 cm to 5.1 cm. Find the approximate increase in area.

11. A fair die is rolled. Consider the following events $A = \{2, 4, 6\}$, $B = \{4, 5\}$ and $C = \{3, 4, 5, 6\}$. Find

(i) $P(A \cup B / C)$ (ii) $P(A \cap B / C)$

12. Show that the determinant value of a skew-symmetric matrix of odd order is always zero.

SECTION - C

13. Solve the following differential equation: $(x+y)^2 \dfrac{dy}{dx} = a^2$.

14. Find the minimum value of n for which $\tan^{-1}\dfrac{n}{\pi} > \dfrac{\pi}{4}, n \in N$.

OR

Show that $\tan\left(\dfrac{1}{2}\sin^{-1}\dfrac{3}{4}\right) = \dfrac{4-\sqrt{7}}{3}$.

15. Find the equation of a curve passing through the point (0, 1), if the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x-coordinate (abscissa) and the product of the x-coordinate and y-coordinate (ordinate) of that point.

16. Evaluate $\int \dfrac{1+x^2}{1+x^4}\, dx$.

OR

Evaluate $\int x.(\log x)^2\, dx.$.

17. Using the properties of determinants, show that $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(z-x)^2$.

OR

Find the value of θ satisfying $\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$.

18. Using properties of definite integrals, evaluate $\int_{\pi/6}^{\pi/3} \dfrac{dx}{1+\sqrt{\tan x}}$.

19. In an activity organized in the school, Rohan was given the task to put the slogan 'Satyamav Jayte' on a trapezium shaped card sheet. If the length of three sides of a trapezium other than base are equal to 10 cm, find the area of the trapezium when it is maximum. Explain the meaning of 'Satyamev Jayte'.

20. Find the coordinates of the point, where the line passes through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

21. A can hit target 4 times out of 5 times, B can hit target 3 times out of 4 times and C can hit target 2 times out of e times.
They fire simultaneously. Find the probability that

(i) any two out of A, B and C will hit the target.
(ii) None of them will hit the target

22. Let $\vec{a} = 2\hat{i} + \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ be three vectors. Find a vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ *and* $\vec{r}.\vec{a} = 0$.

23. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that be guesses. Assuming that, a student who guesses at the answer will be correct with probability 1/4. What is the probability that a student knows the answer given that he answered it correctly?

SECTION - D

24. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is almost half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by almost 600 units. If the company makes profit of `12 and `16 per doll respectively on dolls A and B, then how many of each should be produced weekly in order to maximise the profit?
Why are small scale industries important in India? What values are being promoted by establishing small scale industries?

25. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$ and $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

26. Find the image of the point $(1,6,3)$ on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Also, write the equation of the line joining the given point and its image and find the length of segment joining the given point and its image.

OR

Find the foot of the perpendicular from the point $(0,2,3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also, find the length of the perpendicular.

27. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$.

28. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

OR

Solve the following system of equations by matrix method, where $x \neq 0, y \neq 0$ *and* $z \neq 0$.
$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$ and $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$.

29. Let $A = \{x \in R : 0 \leq x \leq 1\}$. If $f : A \to A$ is defined by $f(x) = \begin{cases} x, & if\ x \in Q \\ 1-x, & if\ x \notin Q \end{cases}$

Then prove that $fof(x) = x$ for all $x \in A$.

OR

Let $A = N \times N$ and * be the binary operation on A defined by $(a,b)*(c,d)=(a+c,b+d)$. Show that * is commutative and associative. Find the identity element for * on A, if any .