

**General Instruction:**

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Question 1-4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5-12 in Section B are short answer type questions carrying 2 marks each.
5. Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.
6. Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.
7. There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

Time- 3 Hours

[Max. Marks-100]

**SECTION - A**

1. Evaluate  $\int x^x (1 + \log x) dx$ .
2. The sides of an equilateral triangle are increasing at the rate of 2 cm/s. How far is the area increasing when the side is 10 cm?
3. Find the non-zero values of  $x$  satisfying the matrix equation.  

$$x \begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2 \begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2 \begin{bmatrix} x^2 + 8 & 24 \\ 10 & 6x \end{bmatrix}$$
4. Find the value of  $\lambda$  such that the line  $\frac{x-2}{12} = \frac{y-1}{\lambda} = \frac{z-3}{-8}$  is perpendicular to the plane  $3x - y - 2z = 7$ .

**SECTION - B**

5. If  $y = |x - x^2|$ , then find  $\frac{dy}{dx}$  at  $x = 1$ .
6. Find the slope and equation of the normal to the curve  $x = 1 - a \sin \theta$ ,  $y = b \cos^2 \theta$  at  $\theta = \frac{\pi}{2}$ .
7. Evaluate  $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$ .
8. Find an angle  $\theta$  which increases twice as fast as its sine.
9. Show that all the positive integral powers of a symmetric matrix are symmetric.
10. Find the point on the parabola  $y^2 = 18x$  at which the ordinate increases at twice the rate of the abscissa.
11. Show that the three points  $A(1, -2, -8)$ ,  $B(5, 0, -2)$  and  $C(11, 3, 7)$  are collinear and also find the ratio in which B divides AC.

12. How many dice must be thrown so that there is a better than even chance of obtaining a six?

### SECTION - C

13. Show that 
$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

OR

If  $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$ , by using properties of determinants, find the value of  $f(2x) - f(x)$ .

14. If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ , then show that  $(1-x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0$ .

15. Find the equation of a curve passing through the point (1, 1). If the tangent drawn at any point P(x, y) on the curve meets the coordinate axes at A and B such that P is the mid-point of AB.

16. Evaluate  $\int \frac{1}{3x^2 + 5x + 7} dx$ .

OR

Evaluate  $\int \frac{xe^x}{(x+1)^2} dx$ .

17. Show that  $\int_0^{\pi/2} f(\sin 2x) \sin x dx = \sqrt{2} \int_0^{\pi/4} f(\cos 2x) \cos x dx$ .

18. Solve the following equation.  $\tan^{-1} \sqrt{x^2 + x} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$ .

19. Solve the following differential equation.  $(1+x^2) \frac{dy}{dx} - 2xy = (x^2+2)(x^2+1)$ ..

When  $x=1$  and  $y=2$ .

If  $y$  is distance and  $x$  is the time,  $\frac{dy}{dx}$  is velocity. John rides a vehical beyond the limit on a highway. What suggestion would you give him to help understands the risks of overlapping?

20. Find the distance from the point (3, 4, 5) to the point, where the line  $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$  meets the plane  $x + y + z = 2$ .

21. Find the probability distribution of number off doublets in three throws of a pair of dice.

22. Show that area of the parallelogram whose diagonals are given by  $\vec{a}$  and  $\vec{b}$  is  $\frac{|\vec{a} \times \vec{b}|}{2}$ . Also, find the area of the parallelogram whose diagonals are  $2\hat{i} - \hat{j} + \hat{k}$  and  $\hat{i} + 3\hat{j} - \hat{k}$ .

23. A letter is known to have come from either 'TATANAGAR' or 'CALCULATE'. On the envelope just two letters 'TA' are visible. What is the probability that the letter has come from  
 (i) TATANAGAR?  
 (ii) CALCUTTA?

OR

If A and B are two independent events such that  $P(\bar{A} \cap B) = \frac{2}{15}$  and  $P(A \cap \bar{B}) = \frac{1}{6}$ , then find  $P(A)$  and  $P(B)$ .

### SECTION - D

24. If  $f : R \rightarrow R$  is a function defined by  $f(x) = 2x^3 - 5$ , then show that the function f is a bijective function.

OR

Consider  $f : R \rightarrow R$  given by  $f(x) = 4x + 3$ . Show that f is invertible and find the inverse of f.

25. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the number using matrix method.

OR

If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ , prove that  $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$ .

26. Find the area off the region bounded by  $y = -1, y = 2, x = y^3$  and  $x = 0$ .

27. A variable plane which remains at a constant distance  $3p$  from the origin cut the coordinate axes at A, B and C. Show that the locus of the centroid of  $\Delta ABC$  is  $x^{-2} + y^{-2} + z^{-2} = p^{-2}$ .

OR

If the lines  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$  and  $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$  intersect, find the value of k and hence, find the equation of the plane containing these lines.

28. A manufacturer considers that mean and women workers are equally efficient, so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B.

To produce 1 unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce 1 unit of B. If A and B are priced at `100 and `120 per unit respectively, then how should he use his resources to maximise the total revenue? Form the above as an LPP and solve it graphically. Also, write what quality of manufacturer reflects here?

29. Find the value of p for which the curves  $x^2 = 9p(9-y)$  and  $x^2 = p(y+1)$  cut each other at right angle.