PAPER:1

General Instruction:

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Question 1-4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5-12 in Section B are short answer type questions carrying 2 marks each.
5. Questions 13-23 in Section C are long answer I type questions carrying 4 marks each.
6. Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.
7. There is not overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

Time- 3 Hours **[Max. Marks-100]**

SECTION -A

1. For what value of k, the matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is a skew symmetric matrix?

2. Evaluate : $\int \frac{1}{\sin^2 x \cos^2 x} dx$

3. If a line makes angles α, β, γ with the positive direction of co-ordinate axes, then write the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

OR

Find the vector equation for the line which passes through the point $(1,2,3)$ and is parallel to the line $\frac{x-1}{-2} = \frac{1-y}{3} = \frac{3-z}{-4}$.

4. Find the value of c in Rolle's theorem for the function: $f(x) = x^3 - 3x$ in $\left[-\sqrt{3}, 0\right]$

SECTION - B

5. If $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$

6. Find the equation(s) of the tangent(s) to the curve $y = \left(x^3 - 1\right)\left(x - 2\right)$ points where the curve intersects the x-axis.

7. If the equation of a line is $x = ay + b; z = cy + d$, then find direction ratios of the line and a point on the line.

8. Write in the simplest form; $\sin^{-1}\left[\frac{x+\sqrt{1-x^3}}{\sqrt{2}}\right], -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

9. Form the differential equation of equation $y = a\cos 2x + b\sin 2x,$ where a and b are constants.

OR

Solve the differential equation $\frac{dy}{dx} = \frac{1+\cos 2y}{1+\cos 2x}$

10. For what values of λ, is the function

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \le 0 \\ 4x+1, & \text{if } x > 0 \end{cases}$$

Is continuous at $x = 0$? What about continuity at $x = 1$?

OR

If $x = \frac{at}{1+t^2}, y = \frac{at^2}{1+t^2}$, find $\frac{dy}{dx}$ at $t = 2$

11. Evaluate: $\int_0^{\frac{\pi}{2}} \log\tan x\, dx$

12. If $A = \begin{bmatrix} 0 & b & -2 \\ 3 & 1 & 3 \\ 2a & 3 & -1 \end{bmatrix}$ is symmetric matrix, find the values of a and b

OR

Solve the matrix equation $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$

SECTION - C

13. Let R be a relation defined on the set of natural numbers N as follow:

$$R = \{(x, y); x \in N, y \in N \text{ and } 2x + y = 24\}$$

Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

14. If a, b, c are positive and unequal, show that the following determinant is negative.

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

15. Evaluate : $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$

OR

Find : $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}} dx$

16. The two adjacent sides of parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

17. If $x\sqrt{1+y}+y\sqrt{1+x}=0,\ (x\neq y),$ then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

OR

If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, then find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

18. Find the shortest distance between the lines $\vec{r}=\left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$

and $\vec{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

19. Evaluate $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$

OR

Evaluate $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$

20. Prove that $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi$

21. Solve the differential equation : $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^3$

22. *A* and *B* throw a pair of dice alternatively. *A* wins the game if he gets a total of 9 and *B* wins if he gets a total of 7. If *A* starts the game, find the probability of winning the game by *B*.

23. Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha}=4\hat{i}+5\hat{j}-\hat{k}$ and $\vec{\beta}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{p}\cdot\vec{q}=21$, where $\vec{q}=3\hat{i}+\hat{j}-\hat{k}$.

SECTION - D

24. Using elementary column operation, find the inverse of the following matrix

$$\begin{vmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

OR

Using matrices, solve the following system of linear equations:

$$x-y+2z=7, 3x+4y-5z=-5, 2x-y+3z=12.$$

25. An open box, with a square base is to be made out of a given quantity of metal sheet of area c^2. Show that the maximum volume of the box is $\frac{c^2}{6\sqrt{3}}$.

OR

The sum of surface areas of a sphere and a cuboid with sides $\frac{x}{3}$, x and $2x$ is constant. Show that the sum of their volumes is minimum if x is equal to the three times the radius of sphere.

26. Using integration, find the area of the region bounded by the line $x-y+2=0$, the curve $x=\sqrt{y}$ and Y axis.

OR

Find the area of the region bounded by the region enclosed by curves.

$$(x-6)^2+y^2=36 \text{ and } x^2+y^2=36$$

27. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ and twice of its y-intercept is equal to three times its z-intercept.

28. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let be the probability that he knows the answer and $\frac{2}{3}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly.

29. There are two types of fertilizers '*A*' and '*B*', '*A*' consists of 12% nitrogen and 5% phosphoric acid 'B' consists of 4% nitrogen and 5% Phosphoric acid. After testing the solid conditions, farmer finds that he need at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If '*A*'costs Rs.10 per kg and 'B' costs Rs.8 per kg, then graphically determine how much of each type of fertilizer should be used so that nutrient requirements are at a minimum cost.