TOPIC: Maxima & Minima (AOD)

1.Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

2. Fine the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16}+\frac{y^2}{16}=1$, with its vertex at one end of the major axis.

3. A wire of length 28 m is to be cut into two piece. One of the two pieces is to made into a square and the other into a circle. What a should be the lengths of two pieces, so that the combined area of circle and square is minimum?

4. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also, show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

5. A window is of the form of a semi – circle with a rectangle on its diameter. The total perimeter of the window is 10 m. find the dimensions of the window to admit maximum light through the whole opening.

6.Find the point p on the curve $y^2 = 4ax$, which is nearest to the point (11 a,0)

7.Prove that the semi – vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.

8. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

9.A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, then find the dimensions of the rectangle that will produce the largest area of the window.

10.Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum areaShow that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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12.Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

13. Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to diameter of base.

14.Prove that radius of right cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

15.An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

16. An open tank with a square base and vertical sides is to be constructed from a metal sheet, so as to hold a given quantity of water. Show that the total surface area is least when depth of the tank is half its width.

17.Prove that the area of a right – angles triangle of given hypotenuse is maximum, when the triangle is isosceles.

18. A tank with rectangular base and rectangular sides, open at the top is to be constructed, so that its depth is 2 m and volume is 8 m^3. If building of tank cost Rs. 70 per sq m for the base and Rs 45 per sq m for sides. What is the cost of least expensive tank?

19.Show that the height of the closed right circular cylinder, of given volume and minimum total surface area, is equal to its diameter.

20.Show that the volume of the greatest cylinder can be inscribed in a cone of height h and semi- vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

21. Show that of all the rectangles of given area, the square has the smallest perimeter.

22.Show that the semi – vertical angle of a right circular cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

23.Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point (1,4).

24. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least, when the side of the square is double the radius of the circle.

25.If the length of three sides of a trapezium other than the base are each equal to 10 cm, then find the area of the trapezium, when it is maximum.

26. Show that the right circular cylinder , open at the top and of given surface area and maximum volume is such that its height is equal to the radius of the base.

27.A manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to reach maximum profit.

28.If the sum of the length of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

29. Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

30.Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of it base.

31. Find the area of the greatest rectangle that can be inscribed in an ellipse.

32. Of all the closed right circular cylindrical cans of volume 128 $\pi\ cm^3$., find the dimensions of the can which has minimum surface area.

33.AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles.

34. If the sum of the length of the hypotenuse and a side of a right – angled triangle is given, then show that the area of the triangle is maximum, when the area of the triangle is maximum, when the angle between them is 60^0.

35.Show that the semi – vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1} 1/\sqrt{3}$.
