Topic: Linear Programming

1. An aeroplane of an airline can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each economy-class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by economy class than by first class. Determine how many of each type of tickets must be sold in order to maximize the profit for the airline. What is the maximum profit?

2. If a young man rides his motorcycle at 25 km per hour, he has to spend Rs 2 per kilometer on petrol; if he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs 5 per kilometre. He has Rs 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it.

3. A furniture dealer deals in only two items; tables and chairs. He has Rs 5000 to invest and a space to store at most 60 pieces. A table costs him Rs 250 and a chair, Rs 50. He can sell a table at a profit of Rs 50 and a chair at a profit of Rs 15. Assuming that he can sell all the items that he buys, how should he invest his money in order that he may maximize his profit?

4. A man owns a field of area 1000 m^2. He wants to plant fruit trees in it. He has a sum of Rs 1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 m^2 of ground per tree and costs Rs 20 per tree, and type B requires 20 m^2 of ground per tree and costs Rs 25 per tree. When full grown, a type – A tree produces an average of 20 kg of fruit which can be sold at a profit of Rs 2 per kg and a type-B tree produces an average of 40 kg of fruit which can be sold at a profit of Rs 1.50 per kg. How many of each type should be planted to achieve maximum profit when trees are fully grown? What is the maximum profit?

5. A medicine company has factories at two places, X and Y. From these places, supply is made to each of its three agencies situated at P, Q and R. The monthly requirements of the agencies are respectively 40 packets, 40 packets and 50 packets of medicines, while the production capacity of the factories at X and Y are 60 packets and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given as follows.

	Transportation cost per packet (in Rs)	
From / to	X	Y
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also, find the minimum cost.

6. A brick manufacture has two depots, P and Q, with stocks of 30000 and 20000 bricks respectively. He receives orders from three buildings, A, B, C, for 15000, 20000 and 15000 bricks respectively. The costs of transporting 1000 bricks to the buildings from the depots are given below.

	Cost of Transportation (in Rs per 1000 bricks)		
From / to	A	B	C
P	40	20	30
Q	20	60	40

How should the manufacture fulfill the orders so as to keep the cost of transportation minimum?

7. A company has factories located at each of the two places P and Q. From these locations, a certain commodity is delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots area respectively, $7, 6$ and 4 units of the commodity while the weekly production capacities of the factories at P and Q are respectively 9 and 8 units. The cost of transportation per unit is given below.
8. A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of $Rs\ 30$ and $Rs\ 25$ per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit?

9. A small firm manufactures items A and B. The total number of items that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be $Rs\ 300$ and that on one unit of item B be $Rs\ 160$, how many of each type of item should be produced to maximize the profit? Solve the problem graphically.

10. A small firm manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and half and hour for a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs 300 and that on a chain is Rs 190, how many of each should be manufactured daily so as to maximize the profit?

11. Suppose every gram of wheat provides $0.1\ g$ of proteins and $0.25\ g$ of carbohydrates, and the corresponding values for rice are $0.05\ g$ and $0.5\ g$ respectively. Wheat costs $Rs\ 5$ and rice $Rs\ 20$ per kilogram. The minimum daily requirements of proteins and carbohydrates for an average man are $50\ g$ and $200\ g$ respectively. In what quantities

should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of proteins and carbohydrates at minimum cost, assuming that both wheat and rice are to be taken in the diet?

12. A hospital dietician wishes to find the cheapest combination of two foods, A and B, that contains at least 0.5 milligram of thiamin and at least 600 calories. Each unit of A contains 0.12 milligram of thiamin and 100 calories, while each unit of B contains 0.10 milligram of thiamin and 150 calories. If each food costs 10 paise per unit, how many units of each should be combined at a minimum cost?

13. To maintain one's health, a person must fulfill certain minimum daily requirements for the following three nutrients – calcium, protein and calories. This diet consists of only items I and II whose prices and nutrient contents are shown below :

	Food I	Food II	Minimum daily requirement
Calcium	10	4	20
Protein	5	6	20
Calories	2	6	12
Price	Re 0.60 per unit	Re 1.00 per unit	

Find the combination of food items so that the cost may be minimum.

14. Minimise and maximise $z = 5x + 10y$
Subject to : $x + 2y \leq 120$,
$x + y \geq 60$,
$x - 2y \geq 0$
$x \geq 0$, $y \geq 0$

15. Maximise $z = -x + 2y$
Subject to constraints :
$x \geq 3$,
$x + y \geq 5$,
$x + 2y \geq 6$,
$y \geq 0$