

TOPIC: DIFFERENTIATION

Page | 1

1. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^\theta (\sin \theta - \cos \theta)$ & $y = ae^\theta (\sin \theta + \cos \theta)$

2. If $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$, then evaluate $\frac{dy^2}{d^2x}$ at $t = \frac{\pi}{3}$.

3. If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

4. If $x = a (\cos t + t \sin t)$ and $y = a (\sin t - t \cos t)$, then find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

5. If $y = \tan^{-1} \left(\frac{a}{x} \right) + \log \sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$

6. If $x = a \sin t (1 + \cos 2t)$ and $y = b \cos t (1 - \cos t)$, then show that at $t = \frac{\pi}{4}$, $\left(\frac{dy}{dx} \right) = \frac{b}{a}$.

7. If $(\tan^{-1} x)^y + y^{\cot x} = 1$, then find $\frac{dy}{dx}$.

8. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

9. If $x \sin(a+y) + \sin a \cos(a+y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

10. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$ or $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$

11. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

12. Prove that : $\frac{d}{dx} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right] = \sqrt{a^2 - x^2}$

13. If $x = a(\theta - \sin \theta)$ & $y = a(1 + \cos \theta)$., then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

Page | 2

14. If $y = (\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$, then find $\frac{dy}{dx}$

15. If $y = \cos^{-1} \left[\frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}} \right]$, then find $\frac{dy}{dx}$.

16. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$.

17. If $y = (x)^x (\sin x)^x$, then find $\frac{dy}{dx}$

18. Differentiate the following function w.r.t.x.

$$(x)^{\cos x} + (\sin x)^{\tan x}$$

19. Differentiate the following function w.r.t.x.

$$(x)^{\sin x} + (\sin x)^{\cos x}$$

20. If $y = (x)^{\cot x} + (\sin x)^x$, then find $\frac{dy}{dx}$

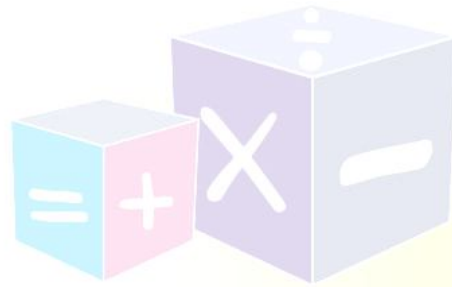
21. If $xy + y^2 = \tan x + y$, then find $\frac{dy}{dx}$.

22. If $y = (\log x)^{\cos x} + \frac{x^2 + 1}{x^2 - 1}$, then find $\frac{dy}{dx}$.

23. If $y = \sin^{-1} \left[\frac{5x + 12\sqrt{1-x^2}}{13} \right]$, then find $\frac{dy}{dx}$.

24. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, ($x \neq y$) , then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

25. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, then find $\frac{dy}{dx}$.



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26. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$

27. If $\sin y = x \sin(a+y)$, then prove that

Page | 3

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

28. If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$, then

show that $\frac{dy}{dx} = \frac{-y}{x}$

29. Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right]$ w.r.t. x.

30. Write the derivative of $\sin x$ with respect to $\cos x$.

31. If $\cos y = x \cos(a+y)$, where $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

32. If $y = \sin^{-1} \{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\}$ and $0 < x < 1$, then find $\frac{dy}{dx}$.

33. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$

34. If $x = 2 \cos \theta - \cos 2\theta$ and $y = 2 \sin \theta - \sin 2\theta$, then prove that $\frac{dy}{dx} = \tan \left(\frac{3\theta}{2} \right)$

35. If $y = (\sin x)^x + \sin^{-1} \sqrt{x}$, then find $\frac{dy}{dx}$.

36. Differentiate the following function with respect to x. $(\log x)^x + x^{\log x}$

37. Show that the function $f(x) = |x-3|$, $x \in \mathbb{R}$, is continuous but not differentiable at $x = 3$.

38. Differentiate the following with respect to x

$$\sin^{-1} \left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right]$$

Page | 4

39. Differentiate $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$ w.r.t. x .

40. If $y = \cos^{-1} \left(\frac{3x + 4\sqrt{1-x^2}}{5} \right)$, then $\frac{dy}{dx}$.

41. Show that the function defined as follows, is continuous at $x = 1$, $x = 2$ but not differentiable at $x = 2$

$$f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$$

42. If $y = (x)^{\sin x} + (\log x)^x$, then find $\frac{dy}{dx}$.

43. If $y = (\log x)^x + (x)^{\cos x}$, then find $\frac{dy}{dx}$.

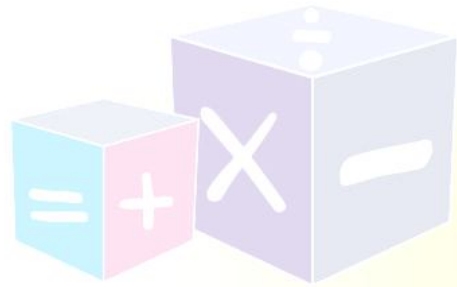
44. If $y = \cot^{-1} \left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right]$,

$0 < x < \frac{\pi}{2}$, then find $\frac{dy}{dx}$.

45. If $y = \sqrt{x^2 + 1} - \log \left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right)$, then find $\frac{dy}{dx}$.

46. Differentiate the following function w.r.t. x .

$$\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right).$$



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47. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

48. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, when $x \neq 0$.

49. If $x = \cos t (3 - 2 \cos^2 t)$ and $y = \sin t (3 - 2 \sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

50. If $(x-y)e^{\frac{x}{x-y}} = a$. Prove that $y \frac{dy}{dx} + x = 2y$.

51. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, when $x \neq 0$.

52. If $y = \log [x + \sqrt{x^2 + a^2}]$, then show that $(x^2 + a^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$

53. If $y = x \log \left(\frac{x}{a+bx} \right)$, then prove that : $x^3 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2$

54. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2 y}{dx^2}$ and $\frac{d^2 y}{dt^2}$.

55. Find $\frac{dy}{dx}$, when $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$

56. If $x = \cos t + \log \tan \frac{t}{2}$ and $y = \sin t$, then find the values of $\frac{d^2 y}{dt^2}$ and $\frac{d^2 y}{dx^2}$ at $t = \frac{\pi}{4}$.

57. If $y = \log [x + \sqrt{x^2 + 1}]$, then prove that $(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$

58. If $\log(\sqrt{1+x^2} - x) = y\sqrt{1+x^2}$, then show that $(1+x^2) \frac{dy}{dx} + xy + 1 = 0$

59. If $x = a(\theta + \sin \theta)$ & $y = a(1 - \cos \theta)$, then find $\frac{d^2 y}{dx^2}$.

60. If $y = a \sin x + b \cos x$, then prove that

$$y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2.$$

61. If $x = a(\cos \theta + \theta \sin \theta)$ & $y = a(\sin \theta - \theta \cos \theta)$ then find $\frac{d^2 y}{dx^2}$.

62. If $y = \cot^{-1} x^2$, then show that $(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$

63. If $y = \operatorname{cosec}^{-1} x$, $x > 1$ then show that

$$x(x^2 - 1) \frac{d^2 y}{dx^2} + (2x^2 - 1) \frac{dy}{dx} = 0$$

64. If $y = e^x (\sin x + \cos x)$, then show that

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$$

65. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then show that

$$(1-x^2) \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - y = 0$$

66. If $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ and $y = a \sin \theta$, then find the value of $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

67. If $y = 3 \cos (\log x) + 4 \sin (\log x)$, then show that

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$



68. If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, then show that

$$(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$$

Page | 7

69. If $y = e^x \sin x$, then prove that

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$$

70. If $x = \tan\left(\frac{1}{a} \log y\right)$, then show that

$$(1+x^2) \frac{d^2 y}{dx^2} + (2x-a) \frac{dy}{dx} = 0$$

71. If $y = (\tan^{-1} x)^2$, then show that

$$(x^2+1)^2 \frac{d^2 y}{dx^2} + 2x(x^2+1) \frac{dy}{dx} = 2$$

72. If $y = P e^{\alpha x} + Q e^{\beta x}$, then show that

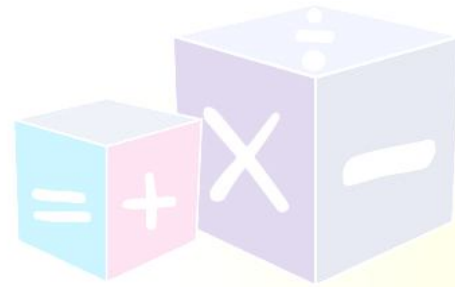
$$\frac{d^2 y}{dx^2} - (a+b) \frac{dy}{dx} + aby = 0$$

73. If $y = x^x$, then prove that

$$\frac{d^2 y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$$

74. If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, then find $\frac{d^2 y}{dx^2}$.

75. If $x = a \sin t$ and $y = a(\cos t + \log \tan(t/2))$, then find $\frac{d^2 y}{dx^2}$



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