

TOPIC: DETERMINANTS

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1. Using properties of determinants, prove that:

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

2. Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1-b^2 & 2ab & -2b \\ 2ab & b^2+1-a^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

3. Prove that:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

4. Show that, if $x \neq y \neq z$ and

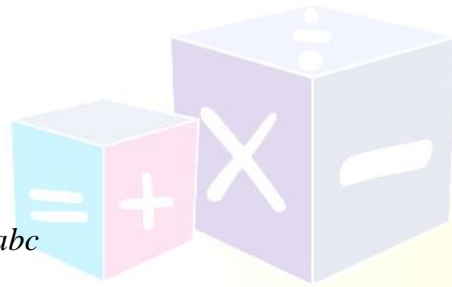
$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \text{ then } 1 + xyz = 0$$

5. Using properties of determinants, prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

6. Using properties of determinants, prove that:

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$



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7. Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

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8. Prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

9. Using properties of determinants, prove that:

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

10. Using properties of determinants, prove that:

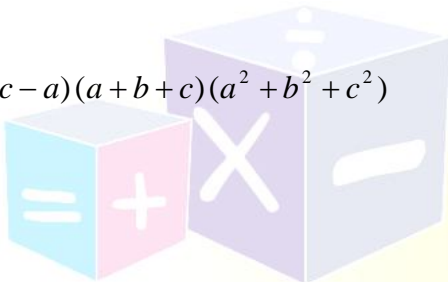
$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

11. Using properties of determinants, prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(xy)(y-z)(z-x)$$

12. Using properties of determinants, prove that:

$$\begin{vmatrix} x+y & 2x & 2x \\ x2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

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Using properties of determinants, prove that:

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$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

14. Using properties of determinants, prove that:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(bc+ca+ab)$$

15. Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

16. Show that $\Delta = \Delta_1$, where

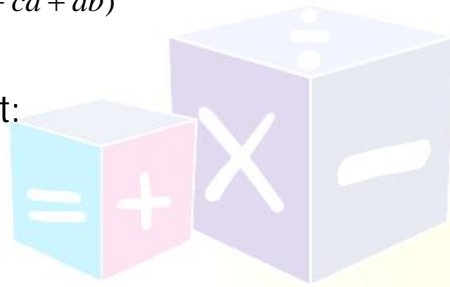
$$\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$$

17. Using properties of determinants, prove that:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

18. Using properties of determinants, prove that:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$



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19. Using properties of determinants, prove that:

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \alpha & \gamma + \alpha & \alpha + \beta \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma)$$

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20. Using properties of determinants, prove that:

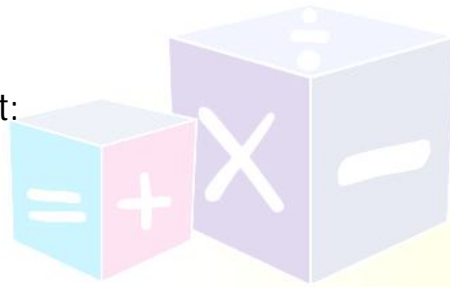
$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz + xy + yz + zx$$

21. Prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$

22. Using properties of determinants, prove that:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$



23. Using properties of determinants, prove that:

$$\begin{vmatrix} a+x & x & x \\ x & a+x & x \\ x & x & a+x \end{vmatrix} = 0$$

24. Using properties of determinants, prove that:

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

25. Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c),$$

26. Using properties of determinants, prove that:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx),$$

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27. Using properties of determinants, prove that:

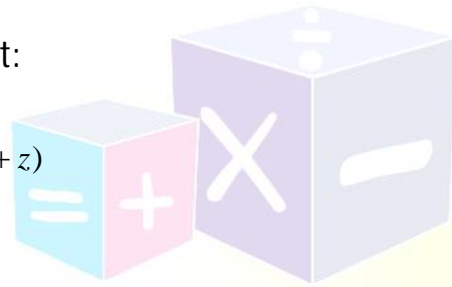
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$$

28. Using properties of determinants, prove that:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

29. Using properties of determinants, prove that:

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$$



30. Prove the following, using properties of determinants:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & 8c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

31. Using properties of determinants, prove that:

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2+z^2$$
