TOPIC: RELATION , FUNCTION & BINARY OPERATIONS

1.If * is a binary operation on Q, defined by a*b = 3ab/5. Show that * is commutative as well as associative. Also, find its identity, if it exists.

2.If A = N X N and * is a binary operation on a defined by (a,b)*(c,d) = (a+c , b+d). Show that * is commutative and associative. Also find identity element for * on A, if any.

3.If * is the binary operation on N given by a*b = LCM of a and b. Find 20*16. Is * (i) commutative and (ii) associative?

4.If * is a binary operation on set Q of rational numbers such that a*b = $(2a-b)^2$, a, b $\in$ Q. Find 3*5, 5*3. Is 3*5 = 5*3?

5.If the binary operation *, defined on Q, is defined as a*b = 2a + b – ab, for all as b $\in$ Q. Find the value of 3*4.

6.If * is a binary operation on set Q of rational numbers defined as a*b = ab/5. Write the identity for *, if any.

7.If S is the set of all rational numbers except 1 and * be defined on S by a*b = a + b – ab, for all a,b $\in$ s.

Prove that,
(a) * is a binary operation on S.
(b) * is commutative as well as associative.

8.Consider the binary operations * : R X R $\rightarrow$ R and o : R X R $\rightarrow$ R defined as a*b = $|a-b|$ and a o b = a. For all a,b $\in$ R. Show that * is commutative but not associative, 'o' is associative but not commutative.

9.Let * : R X R $\rightarrow$ R given by (a, b) $\rightarrow$ a + $4b^2$ be a binary operation. Compute (-5)*(2*0)

10.Let * is binary operation on the set of all non-zero real numbers, given by a*b=ab/5 for all a, b $\in$ R {0}. Find the value of x, given that 2*(x*5) = 10.

11.Let * is a binary operation on N given by a*b = LCM (a,b) for all a,b $\in$N. Find 5*7

12.Let * : R X R $\rightarrow$R is defined as a*b = 2a + b. Find (2*3)*4.

13.If the binary operation * on the set of integers Z, is defined by a*b = a + 3b^2, then f Consider the binary operation * on the set {1,2,3,4,5} defined by a*b = min{a,b}. Write operation table of operation *.

14.A binary operation * on the set {0,1,2,3,4,5} is defined as a*b

$$\begin{cases} a + b, & if\ a + b < 6 \\ a + b - 6, & if\ a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element and the value of 8*3.

15.Let * is a binary operation on set of integers I defined by a*b = 3a + 4b – 2, then find the value of 4*5.

16.Let * is a binary operation on set of integers I, defined by a*b = 2a + b – 3. Find value of 3*4.

17.If the binary operation * on set of integers Z is defined by a*b = a + 3b^2, then find the value of 2*4.

18.Let * is the binary operation on N given by a*b = HCF (a,b) where, a, b $\in$ N. Write the value of 22*4.

19.If f:N $\rightarrow$N is defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$$

Find whether the function f is bijective.

20.Show that relation R in the set of real numbers, defined as R = {(a,b) : a $\leq b^2$} is neither reflexive , nor symmetric nor transitive.

21.If the function f : R $\rightarrow$ R is given by

f(x) = x2 + 3x + 1 and g : R $\rightarrow$R is given by
g(x) = 2x – 3 then find (i) fog and (ii) gof.

22.If the function f : R $\rightarrow$R is given by

f(x) = $\frac{x+3}{3}$ and g : R $\rightarrow$R is given by
g(x) = 2x – 3, then find
i) fog and (ii) gof. Is f^{-1} = g ?

23. If A = {1,2,3,...,9} and R be the relation in A X A defined by (a,b) R (c,d). If a + d = b + c for (a,b), (c,d) in A X A. Prove that R is an equivalent relation, Also, obtain the equivalent class [(2,5)].

24.If the function f:R $\rightarrow$R is given by f(x) = x^2+2 and g : R $\rightarrow$R is given by g(x) = $\frac{x}{x-1}$; $x \neq 1$, then fog and gof and hence, find fog(2) and gof (-3).

25.If A = R – {2} and B = R – {1}. If : A $\rightarrow$B is a function defined by f(x) = $\frac{x-1}{x-2}$, then show that f is one-one and onto. Hence, f^{-1}.

26.Show that the function f in

A = R $-\left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

27.Consider f : R_+ $\rightarrow$[4, ∞] given by f(x) = x^2 + 4. Show that f is invertible with the inverse f^{-1} of f given by f^{-1}(y) = $\sqrt{y-4}$, where R_+ is the set of all non – negative real numbers.

28.Show that f : N $\rightarrow$N, given by

f(x) = $\begin{cases} x+1, & if\ x\ is\ odd \\ x-1, & if\ x\ is\ even \end{cases}$ is bijective (both one-one and onto).

29. If R = {(a, a^3); is a prime number less than 5} be a relation. Find the range of R.

30.If f:{1,3,4} $\rightarrow${1,2,5} and g: {1,2,5}$\rightarrow${1,3} given by f = {{1,2), (3,5), (4,1)} and g = {(1,3), (2,3), (5,1)}.

Write down gof.

31.Let R is the equivalent relation in the set A = {0,1,2,3,4,5} given by R = {(a,b): 2 divides (a – b)}. Write the equivalence class [0].

32.If R = {(x,y); x + 2y = 8} is a relation on N, then write the range of R.

33.If A = {1,2,3}, B = { 4,5,6,7} and f = {(1,4), (2,5), (3,6)} is a function from A to B. State whether f is one-one or not.

34.If If f:R$\rightarrow$R defined by $f(x)=\frac{3x+5}{2}$ is an invertible function, then find $f^{-1}(x)$.

34.State whether the function f:N$\rightarrow$N given by f(x) = 5x is injective, surjective or both.

35.If f : R $\rightarrow$R defined by $f(x)=\frac{2x-7}{4}$ is an invertible function, then find $f^{-1}(x)$.

36.If f : W $\rightarrow$W, is defined as f(x) = x – 1 , if x is odd and f(x) = x + 1, if x is even. Show that f is invertible. Find the inverse of f, where W is the set of all whole numbers. f : R$\rightarrow$R is defined by f(x) = 3x + 2, then define f[f(x)].

37. Show that the relation S in set

A = {x$\in$Z : 0 $\leq$ x $\leq$ 12} given by
S = {(a,b) : a,b $\in$Z, $|a-b|$ is divisible by 4} is an equivalent relation. Find the set of all elements related to A.

38.Show that the relation S defined on set N X N by (a,b) S (c,d) $\Rightarrow$a + d = b + c is an equivalence relation.

39.Consider f : $R_+ \rightarrow [-5, \infty]$ given by f(x) = $9x^2 + 6x - 5$, show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.

40.If f : X $\rightarrow$T is a function. Define a relation R on X given by R = {(a,b) : f(a) = f(b)}. Show that R is an equivalence relation on X.

41.Show that a function f : R $\rightarrow$R given by f(x) = ax + b, a , b $\in$ R, a $\neq$ o is a bijective.

42.Prove that the relation R in set

A = (1,2,3,4,5} given by R = {(a,b) : $|a-b|$ is even } is an equivalent relation.

43. Write fog, if R$\rightarrow$R and g: R $\rightarrow$ R are given by f(x) = $|x|$ and g(x) = $|5x-2|$.

44.Write fog, if R $\rightarrow$R and g:R $\rightarrow$R are given by f(x) = $8x^3$ and g(x) = $x^{1/3}$.

45.State the reason for the relation R in the set {1,2,3} given by R = {(1,2), (2,1)}. Not to be transitive.

46.What is the range of the function

$f(x)=\frac{|x-1|}{x-1}, x\neq 1?$

47.If f : R $\rightarrow$R is defined by f(x) = $(3-x^3)^{1/3}$, then

find fof(x).

48.If f is an invertible function, defined as

$f(x)=\frac{3x-4}{5}$, then write $f^{-1}(x)$.

49.If f:R$\rightarrow$R and g:R$\rightarrow$R are given by f(x) = sinx and g(x) = $5x^2$, then find gof(x).

50.If f(x) = $27x^3$ and g(x) = $x^{1/3}$, then find gof(x).

51.If the function f : R $\rightarrow$R, defined by f(x) = 3x – 4 is invertible, then find f^{-1}.

52. If $f:R \to R$ is defined as $f(x) = 10x + 7$. Find the function $g : R \to R$, such that $gof = fog = I_R$.

53. Show that the function $f : W \to W$ defined by

$$f(n) = \begin{cases} n+1, & if\ n\ is\ even \\ n-1, & if\ n\ is\ odd \end{cases}$$

54.If $f : R \to R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijection.

55.If Z is the set of all integers and R is the relation on Z defined as

R = {(a,b):a,b $\in$ Z and a – b is divisible by 5}. Prove that R is an equivalent relation.

56. If $f,g : R \to R$ are two functions defined as $f(x) = |x| - x, \forall x \in R$, then find fog and gof.

57.If R is a relation defined on the set of natural numbers N as follows:

R = {(x,y), $x \in N, Y \in N$ and 2x + y = 24}, then find the domain and range of the relation R. Also, find if R is an equivalent relation or not.

58.If A = R – {3} and B = R – {1}.

Consider the function $f:A \to B$ defined by $f(x) = \frac{x-2}{x-3}$ for all $x \in A$. Then show that f is bijective. Find $f^{-1}(x)$.

59. Show that the relation S in the set R of real numbers defined as S = {(a,b) : a,b $\in$ R and a $\leq b^3$} is neither reflexive nor symmetric nor transitive.
