# TOPIC: 3D, LINE & PLANE



Q. 1 If a unit vector $\hat{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle $\theta$ with $\hat{k}$, then find the value of $\theta$.

Q. 2 Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

Q. 3 If a line has direction ratios 2, –1, –2, then what are its direction cosines?

Q. 4 What are the direction cosines of a line which makes equal angles with the coordinate axes?

Q. 5 Write the distance of a point P (a, b, c) from X – axis.

Q. 6 If the Cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, the n write the vector equation for the line.

Q. 7 Write the equation of the straight line through the point $(\alpha, \beta, \gamma)$ and parallel to Z – axis.

Q. 8 Find the direction cosines of the line

$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

Q. 9 The equation of line is

$\frac{2x-5}{4} = \frac{y+4}{3} = \frac{6-z}{6}$

Find the direction cosines of the line parallel to this line.

Q. 10 Find the direction cosines of the line parallel to this line.

$\frac{4-x}{2} = \frac{y+3}{5} = \frac{z+2}{6}$.

Write the direction cosines of the line parallel to above line.

Q. 11 If P = (1,5,4) and Q = (4,1,-2) , then find the direction ratios of PQ.

Q. 12 Find the equation of line passing through points A (0,6, –9) and B (–3, –6,3). If D is the foot of perpendicular drawn from the point C ( 7,4, –1) on the line AB, then the coordinates of point D and equation of line CD.

Q. 13 Find the image of the point (1,6,3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, write the equation of the line joining the given points and its image and find the length of segment joining given point and its image.

Q. 14 Write the vector equations of following lines and hence find the distance between them

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$, $\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$

Q. 15 The points A(4,5,10) , B(2,3,4) and C( 1,2, $-1$) are three vertices of parallelogram ABCD. Find the vector equations of sides AB and BC and also find coordinates of point D.

Q. 16 Find the coordinates of foot of perpendicular drawn from the point (0,2,3) on the line

$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Also, find the length of perpendicular.

Q. 17 Show that the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ intersect. Also, find their point of intersection.

Q. 18 Find the direction cosines of the line $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$. Also, find the vector equation of the line through the point A (–1,2,3) and parallel to the given line.

Q. 19 Find the angle between the lines

$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and

$\vec{r}=7\hat{i}-6\hat{j}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$

Q. 20 Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also, find their point of intersection.

Q. 21 Find the value of p, so that the lines

$l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also, find the equation of a line passing through a point (3, 2, –4) and parallel to line $l_1$.

Q. 22 Write the vector equation of the line given by $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

Q. 23 Equation of line is $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$

Find the direction cosines of a line parallel to above line.

Q. 24 If the equations of line AB is

$\frac{3-x}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$,

then write the direction ratios of the line parallel to above line AB.

Q. 25 Find the distance of point (2,3,4) from X – axis.

Q. 26 Write the equation of line parallel to the line

$\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through point (1,2,3).

Q. 27 Write the direction cosines of a line parallel to the line

$\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$

Q. 28 Find shortest distance between lines

$\vec{r} = (1+2\lambda)\hat{i} + (1-\lambda)\hat{j} + \lambda\hat{k}$

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Q. 29 Find the value of $\lambda$, so that following lines are perpendicular to each other

$\frac{x+5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$

and

$\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$

Q. 30 Find the value of $\lambda$, so that lines $\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x+1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ are perpendicular to each other.

Q. 31 Find the value of $\lambda$, so that lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other.

Q. 32 Find the distance of the point P (–1, –5, –10) from the point of intersection of the line joining the points A (2, –1, 2) and B (5,3,4) with the plane x – y + z = 5.

Q. 33 Find the vactor and cartesian forms of the equation of the plane passing through the point (1,2, –4) and parallel to the lines.

$\vec{r}=1+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

$\vec{r}=(1\hat{i}-3\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$

Also, find the distance of the point (9 ,–8, –10) from the plane thus obtained.

Q. 34 Find the perpendicular distance of the point (2,3,4) from the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find coordinates of foot of perpendicular.

Q. 35 Find the shortest distance between lines whose vector equations are

$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$

$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

Q. 36 Find shortest distance between the lines

$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ *and*

$\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$

Q. 37 Find the equation of the perpendicular from point (3, –1, 11) to line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also, find the coordinates of foot of perpendicular and the length of perpendicular.

Q. 38 Find the perpendicular distance of point (1,0,0) from lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of foot of perpendicular and equation of perpendicular.

Q. 39 Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P (1,3,3).

Q. 40 Find the shortest distance between the lines

$l_1: \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{1}$

$l_2: \frac{x-2}{2}=\frac{y+1}{1}=\frac{z+1}{2}$

Q. 41 Find the shortest distance between the following lines:

$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}, \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

Q. 42 Find the distance between the lines $l_1$ and $l_2$ given by

$l_1: \vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

$l_2: \vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

Q. 43 Find the vector and Cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$, *and* $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

Q. 44 The cartesian equation of line is 6x – 2 = 3y + 1 = 2z – 2 . Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, –1, –1) which are parallel to the line.

Q. 45 Find the shortest distance between the two lines whose vector equations are

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

and $\vec{r} = (-4\hat{i} - 1\hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

Q. 46 A line passes through the point (2, –1,3) and is perpendicular to the lines

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

and $\vec{r} = 2\hat{i} - \hat{j} - 3\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and Cartesian forms.

Q. 47 Find the shortest distance between the lines whose vector equations are

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

and $\vec{r} = 2\hat{i} + 1\hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Q. 48 Find the shortest distance between the two lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(1\hat{i} - 3\hat{j} + 2\hat{k})$

and $\vec{r} = (4\hat{i} + 5\hat{j} + 1\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 1\hat{k})$

Q. 49 Show that the lines

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

are intersecting. Hence, find their point of intersection.

Q. 50 Find the vector and cartesian equations of line passing through point (1,2, –4) and perpendicular to two lines.

$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$

and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$



Q. 51 Find the angle between following pair of lines

$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$

and $\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$

and check whether the lines are parallel or perpendicular.

Q.52. Write the vector equation of the plane passing through the point (a,b,c) and parallel to the pane $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=2$

Q.53Find the length of the perpendicular drawn from the origin to the plane 2x – 3y + 6z + 21 = 0.

Q.54. Find distance of the plane 3x – 4y + 12z = 3 from the origin.

Q.55.Write the intercept cut-off by plane 2x + y – z = 5 on X – axis.

Q.56.Write the distance of following plane from origin , 2x – y + 2z + 1 = 0.

Q.57.Find the value of , such that $\lambda$ the line $\frac{x-1}{6}=\frac{y-1}{\lambda}=\frac{z+5}{-4}$ is perpendicular to the plane 3x – y – 2z = 7.

Q.58. A plane makes intercept -6, 3, 4 respectively on the coordinates axes. Find the length of the perpendicular from the origin on it.

Q.59. Find the distance of the point (– 1, –5, –10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda.\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Q.60. Find the coordinates of the foot of perpendicular and the perpendicular distance of point P(3,2,1) from the plane 2x – y + z + 1 = 0. Also, image of the point in the plane.

Q.61.Find the distance of the point (2,3,4) from the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ measured parallel to the plane 3x+2y+2z – 5 = 0.



Q.62. Find the distance of point (–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x + 12y – 3z + 1 = 0.

Q.63.Find the coordinates of image of point (1,3,4) in the plane 2x – y + z+ 3 = 0.

Q.64.From the point P(1,2,4) a perpendicular is drawn on the plane 2x + y – 2z + 3 = 0. Find the equation, the length and the coordinates of foot of perpendicular.

Q.65.Find the vector equation of the plane passing through the three points with position vectors $\hat{i}+\hat{j}-2\hat{k}.\ 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$.

Q.66.Find the equation of plane determined by points A(3, –1,2), B(5,2,4) and C(–1, –1,6) and hence find the distance between planes and point (6,5,9).

Q.67.Find the length and foot of perpendicular from point P(7,14,5) to plane 2x+4y – z = 2. Also, find the image of point P in the plane.

Q.68.Find the equation of plane which contains the line intersections of planes $\vec{r}\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0,\ \vec{r}\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0,$ and which is perpendicular to plane $\vec{r}\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$

Q.69.Find the equation of plane passing through the line of intersection of planes 2x+y – z = 3 and 5x –3y+4z + 9 = 0 and parallel to line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

Q.70. Find the equation of plane passing through the point (–1, –1, 2) and perpendicular to each plane 2x + 3y – 3z = 2 and 5x – 4y + z = 6.

Q.71. Find the equation of plane passing through points (3,4,1) and (0,1,0) and parallel to line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$.



Q.72. Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{3}$ are coplanar.

Q.73. Find the image of the plane having position vector $\hat{i}-3\hat{j}+4\hat{k}$ in the plane

$$\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$$

Q.74. Find the vector equation of the plane through the points (2, 1, –1) and (–1, 3, 4) and perpendicular to the plane x –2y+4z = 10.

Q.75.Find the coordinates of the point , where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersect the plane x –y + z –5 = 0. Also, find the angle between the line and the plane.

Q.76. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.



Q.77. Show that the lines $\vec{r}=.\left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=.\left(4\hat{i}-\hat{k}\right)+\mu(2\hat{i}+3\hat{k})$ are coplanar . Also , find the equation of the plane containing them.

Q.78.Find the distance of the point (1, –2,3) from the plane x–y+z = 5 measured parallel to the line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{-6}$.

Q.79.Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$, whose perpendicular distance from origin is unity.

Q.80.Find the vector equation of the line passing through the point (1,2,3) and parallel to the planes

$\vec{r}\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$

Q.81. Find the coordinates of the point, where the line through (3, –4, –5) and (2, –3, 1) crosses the plane, passing through the point (2,2,1) , (3,0,1) and (4, –1,0).

Q.82. Find the distance between the point (7,2,4) and the plane determine the point A(2,5, –3) , B(–2, –3,5) and C(5,3, –3).

Q.83.Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5, which is perpendicular to the plane x – y + z = 0. Also, find the distance of the plane obtained above, from the origin.

Q.84.Find the distance of the point (2,12, 5) from the point of intersection of the line.

$\vec{r}=2\hat{i}-4\hat{j}+2+\hat{\lambda}.\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-2\hat{j}+\hat{k}\right)=0$

Q.85.Find the equation of the plane that contains the point (1, –1,2) and is perpendicular to both the planes 2x + 3y –2z = 5 and x+2y–3z = 8. Hence, find the distance of point P(–2,5,5) from the plane obtained above.

Q.86.Find the equation of plane that contains the point (1, –1, 2) and is perpendicular to each of planes 2x + 3y – 2z = 5 and x + 2y – 3z = 8.

Q.87.Find the coordinates of point, where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane x + y + 4z = 6.

Q.88.Find the equation of the plane passing through the line of intersection of the plane $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to X-axis.

Q.89.Find the equation of planes(s) passing through the intersection of planes x + 3y + 6 = 0 and 3x – y – 4z = 0 and whose perpendicular distance from origin is unity.

Q.90.Find the equation of plane passing through the point A(1,2,1) and perpendicular to the line joining points P(1,4,2) and Q(2,3,5). Also, find distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$.

Q.91.Find the Cartesian equation of the plane passing through points A(0,0,0) and B(3, –1,2) and parallel to the line

$$\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$$

Q.92.Find the vector and Cartesian equation of a plane containing the two lines.

$\vec{r}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$

Also, show that the line

$\vec{r}=\left(2\hat{i}+5\hat{j}+2\hat{k}\right)+P\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ lies in the planes.



Q.93.Find the equation of plane passing through the point (1,2,1) and perpendicular to line joining points (1,4,2) and (2,3,5). Also, find the coordinates of foot of the perpendicular and the perpendicular distance of the point (4,0,3) from above found plane.

Q.94.Find the equation of plane passing through point P(1,1,1) and containing the line

$$\vec{r}=\left(-3\hat{i}+\hat{j}+5\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}-5\hat{k}\right)$$

Also, show that plane contains the line

$$\vec{r} = \left( -\hat{i} + 2\hat{j} + 5\hat{k} \right) + \mu \left( \hat{i} - 2\hat{j} - 5\hat{k} \right)$$



Q.95.Find the equation of plane passing through the point (–1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 5.

Q.96.Find the vector equation of plane passing through the point A(2,2, –1) , B(3,4,3) and C(7,0,6). Also, find the Cartesian equation of plane.

Q.97.Find the equation of plane passing through point (1,1, –1) and perpendicular to planes x+2y+3z–7=0 and 2x–3y+4z = 0.

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